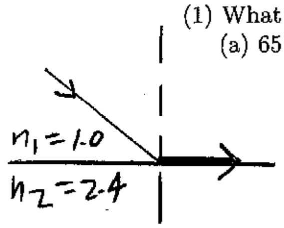
Phys 4C Mid-Term Exam 1 SOLUTIONS – 2025 Fall

(1) What is the critical angle for light going from air ($n = 1.0$) into diamond ($n = 2.4$)?
(a) 65.4 (b) 24.6 (c) 2.4 (d) 76.3 (e) The critical angle is undefined.



$n_1 \sin\theta_1 = n_2 \sin\theta_2$ Find – $\theta_1 = \theta_{critical}$, for which $\theta_2 = 90°$.

For total internal reflection,
$\theta_2 \geq 90°$, but here,

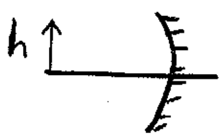
$\theta_1 = \arcsin(n_2/n_1)$
$= \arcsin(2.4/1.0)$.

But $\arcsin(x)$ for $x > 1$ is undefined.

$\boxed{\theta_1 \text{ is undefined} \Rightarrow \text{choice (e)}}$

(2) A concave mirror has a focal length of 7.0 cm. This mirror forms an image that is upright and five times the object height. Determine the object distance (in cm)
(a) −140 (b) −42.0 (c) −28.0 (d) 14.0 (e) 21.0

concave mirror: $f = +7.0$ cm Find – q



$M \equiv \frac{h'}{h} = +5 > 0$ since upright.

$M = +5 = -\frac{q}{p}$

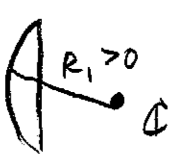
$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$

$\frac{-M}{q} + \frac{1}{q} = \frac{1}{f}$

$(-M+1)f = q$
$(-5+1)(7.0\text{cm}) = q$
$(-4)(7.0\text{cm}) = \boxed{-28.0\text{ cm} = q \Rightarrow \text{choice (c)}}$

(3) A lens is made of glass with $n = 1.5$. Its front surface is convex, with a radius of curvature of 0.10 m. Its back surface is flat. What is the focal length of the lens, in meters?
(a) 0.20 (b) 0.30 (c) 0.40 (d) 0.10 (e) 0.50

Find – f for a lens with $R_1 = +0.10$ m
and $R_2 \to \infty$ since flat



The lensmaker's equation is:

$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \cancel{\frac{1}{R_2}}\right]$ $= 0$

$\frac{1}{f} = (1.5-1)\left[\frac{1}{+0.10\text{m}}\right]$

$f = \frac{0.10\text{m}}{0.5} = \boxed{f = +0.20\text{ m} \Rightarrow \text{choice (a)}}$

(4) A light ray with a frequency of 3.39×10^{14} Hz in air shines with normal incidence (in other words, straight down) into water ($n = 1.33$). The wavelength of the light after it enters the water is (in nanometers): (a) 798 (b) 500 (c) 665 (d) 484 (e) 376

Find – λ_n in water, with $n = 1.33$

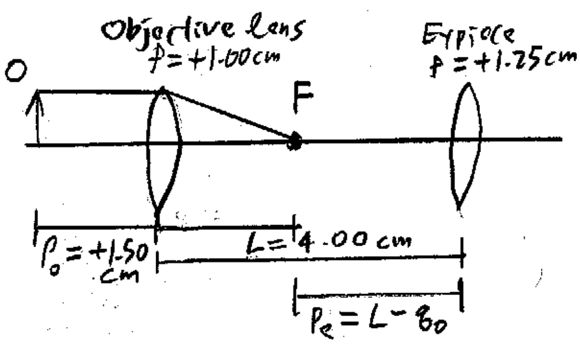
$$n = \frac{\lambda_0}{\lambda_n}$$

$$\lambda_n = \frac{\lambda_0}{n} = \frac{c}{nf} \quad \text{since } \lambda_0 = \frac{c}{f} \text{ in the water.}$$

$$\lambda_n = \left[\frac{(3.00\times10^8\ \cancel{m}/\cancel{s})}{(1.33)(3.39\times10^{14}/\cancel{s})}\right]\left(\frac{10^9\ nm}{1\ \cancel{m}}\right) = \boxed{\lambda_n = 665\ nm \Rightarrow \text{Choice (c)}}$$

(5) A microscope is made of two lenses. The one in front is called the objective lens, and the one in back is called the eyepiece. The objective lens has $f_0 = +1.00$ cm, and the eyepiece has $f_e = +1.25$ cm. The two lenses are separated by a distance of 4.00 cm. If an object is 1.50 cm in front of the objective lens, where (in cm) will the final image from the eyepiece be located?
(a) −5.00 (b) −9.00 (c) −10.00 (d) −12.00 (e) −23.00

Find – q_e



$$\frac{1}{p_o}+\frac{1}{q_o}=\frac{1}{f_o}$$

$$\frac{1}{q_o}=\frac{1}{f_o}-\frac{1}{p_o}=\left(\frac{1}{+1.00cm}\right)-\left(\frac{1}{+1.50cm}\right)=\left(\frac{1.50-1}{+1.50cm}\right)$$

$$=\frac{0.5}{1.50cm} \Rightarrow q_o = 3.00\ cm$$

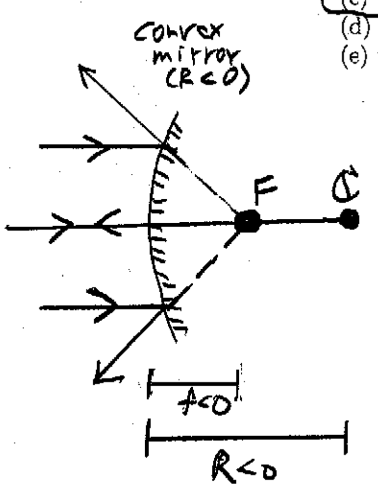
$$p_e = L - q_o = 4.00\ cm - 3.00cm = p_e = +1.00cm$$

$$\frac{1}{p_e}+\frac{1}{q_e}=\frac{1}{f_e}$$

$$\frac{1}{q_e}=\frac{1}{f_e}-\frac{1}{p_e}=\left(\frac{1}{1.25cm}\right)-\left(\frac{1}{1.00cm}\right)=\frac{1-1.25}{+1.25cm}$$

$$\frac{1}{q_e}=\frac{-0.25}{+1.25cm} \qquad \boxed{q_e=-5.00cm \Rightarrow \text{choice (a)}}$$

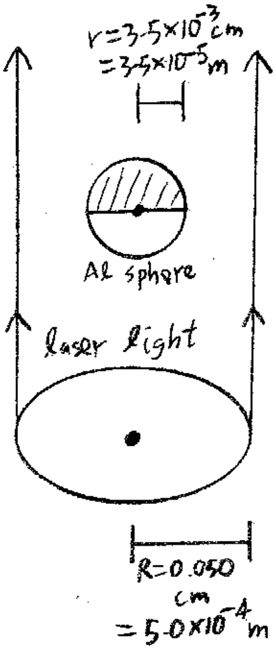
(6) The image formed by a convex mirror (with $R < 0$) is always:
(a) real, magnified, inverted
(b) real, diminished, inverted
(c) virtual, diminished, upright
(d) real, upright
(e) virtual, inverted



By the sign conventions for mirrors, an upright image has $M>0$.
$M=-\frac{q}{p}>0$, since $q<0$ and $p>0$, so the image formed by a convex mirror is <u>always upright</u>.

The image is <u>virtual</u> since it is formed in <u>back</u> of the mirror, as shown.

A diminished image has $|M|<1$, which means $0<M<1$.
All convex mirrors have $f<0$, so they have $1/f<0$,
so: $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}<0$. This implies $\frac{1}{p}<-\frac{1}{q}$,
so: $q/p<-1$, and: $-\frac{q}{p}<+1$.
Since $M=-q/p$, $M<+1$.
From the figure, $M>0$, so $0<M<1$, and so: $\boxed{|M|<1}$, so the image is <u>always diminished</u>.



Box your final answer. No work = No credit on this part.

Problems.

(A) A laser with a circular cross section is used to suspend a spherical aluminum bead in Earth's gravitational field, by balancing the force coming from the radiation pressure against the downward gravitational force. The bead has a mass of 1.0×10^{-6} grams and a radius of 3.5×10^{-3} cm. Some helpful information is:

$g = 9.80665$ m/s^2; $F_{\text{gravity}} = mg$; Volume of a sphere $= \frac{4}{3}\pi r^3$; Area of a circle $= \pi r^2$.

(i) What is the radiation intensity (in W/m^2) needed to support the aluminum bead, assuming the bead reflects all the incident laser light?

(ii) What is the radiation intensity (in W/m^2) needed to support the aluminum bead, assuming the bead absorbs all the incident laser light?

(iii) If the laser beam has a radius of 0.050 cm, what is the power (in Watts) required for this laser?

(iv) What is the maximum magnetic field (in T) of this laser light?

(v) What is the maximum electric field (in V/m) of this laser light?

$F_{gravity} = mg = (1.0\times10^{-6} g)\left(\frac{1 kg}{10^3 g}\right)\left(9.80665 \frac{m}{s^2}\right) = 9.8\times10^{-9}$ kg m/s^2.

This force must balance the force upward from the radiation pressure from the laser.

Since pressure = force/area, $P_{rad} = \frac{F_{gravity}}{A} = \frac{mg}{\pi r^2}$, since the aluminum sphere reflects the light shining on its cross-sectional area, $A = \pi r^2$.

(i) For complete reflection, $P_{rad} = 2I/c$,

so $I_r = \frac{c P_{rad}}{2} = \frac{c}{2}\left(\frac{mg}{\pi r^2}\right) = \frac{(3.00\times10^8 \frac{m}{s})(9.8\times10^{-9} \text{ kg m/s}^2)}{(2\pi)(3.5\times10^{-5} m)^2}$

$\boxed{I_r = 3.8\times10^8 \text{ W/m}^2}$.

(ii) For complete absorption, $P_{rad} = I/c$, so $I_a = c P_{rad} = \boxed{I_a = 7.6\times10^8 \text{ W/m}^2} = 2 I_r$

(iii) I = Power/Area, so Power $= I(\pi r^2)$.

For complete reflection, Power $= I_r \pi (5.0\times10^{-4} m)^2 = 300$ W.

For complete absorption, Power $= I_a \pi (5.0\times10^{-4} m)^2 = 600$ W,

so the power required is the greater of these, $\boxed{\text{Power} = 600 W}$.

(iv) $I = \frac{E_{max} B_{max}}{2\mu_0} = \frac{c B_{max}^2}{2\mu_0}$, since $E_{max}/B_{max} = c$.

$B_{max} = \sqrt{2\mu_0 I/c} = \sqrt{2(4\pi\times10^{-7} \text{ Tm/A})(7.6\times10^8 \text{ W/m}^2)/(3.00\times10^8 \text{ m/s})}$

$\boxed{B_{max} = 2.5\times10^{-3} T}$ for the maximum B_{max}, since $I_a = 2I_r > I_r$.

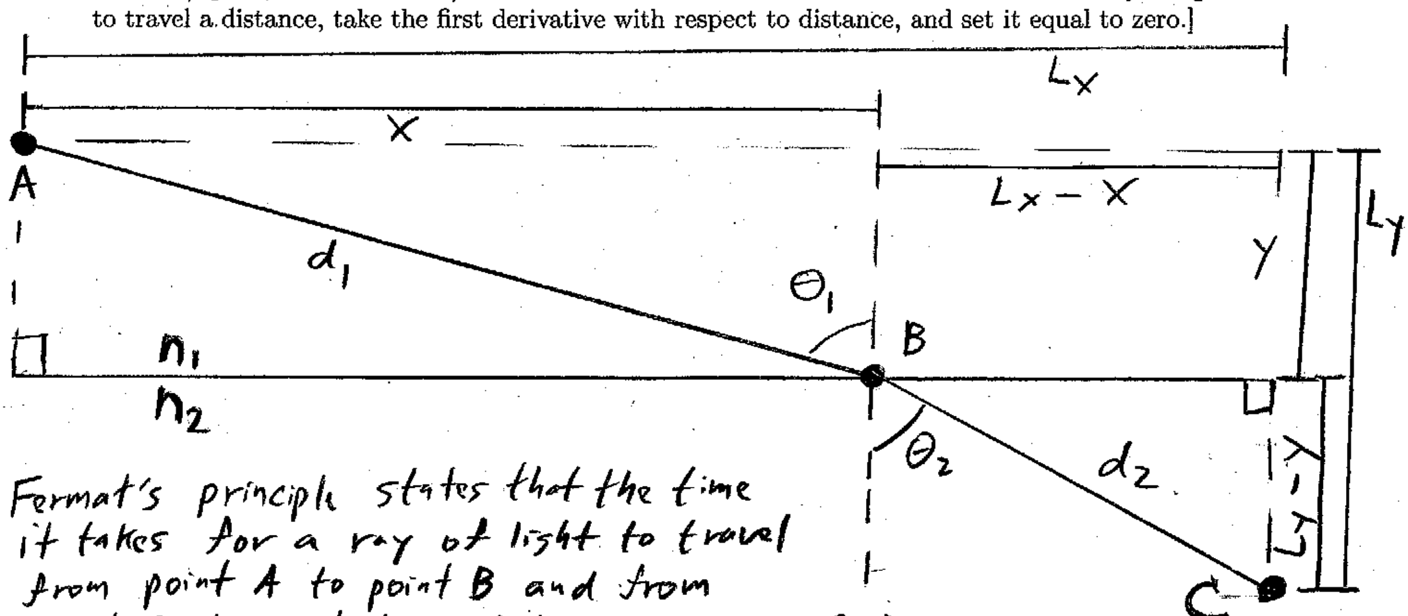
(v) $\frac{E_{max}}{B_{max}} = c$, so $E_{max} = c B_{max} = (3.00\times10^8 \frac{m}{s})(2.5\times10^{-3} T)$

$\boxed{E_{max} = 7.6\times10^5 \text{ V/m}}$.

Alternatively,

$E_{max} = \sqrt{2\mu_0 c I} = 7.6\times10^5$ V/m. ✓

(B) A ray of light travels from point A to point B in a medium with index of refraction n_1, and then from point B to point C in another medium that has index of refraction n_2. Fermat's principle states that this ray of light travels from point A to point B, and then from point B to point C, in the minimum possible time. Use the figure on this page and a little geometry, trigonometry, and calculus to show that this implies Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$. [Hint: recall that time = distance/speed, and that $n = c/v$. Recall also that to find the minimum time it takes a ray of light to travel a distance, take the first derivative with respect to distance, and set it equal to zero.]



Fermat's principle states that the time it takes for a ray of light to travel from point A to point B and from point B to point C must be a minimum, so:

$$t_{total} = t_{AB} + t_{BC} = \frac{d_1}{v_1} + \frac{d_2}{v_2} = \frac{n_1 d_1}{c} + \frac{n_2 d_2}{c}, \text{ since } n \equiv \frac{c}{v}.$$

$$t_{total} = \left[\frac{n_1\sqrt{x^2+y^2}}{c} + \frac{n_2\sqrt{(L_x-x)^2+(L_y-y)^2}}{c}\right], \text{ since: } d_1^2 = x^2+y^2 \text{ and: } d_2^2 = (L_x-x)^2+(L_y-y)^2$$

To find the minimum,

$$\frac{dt_{total}}{dx} = 0 = \left[\frac{2xn_1}{c\sqrt{x^2+y^2}} - \frac{2(L_x-x)n_2}{c\sqrt{(L_x-x)^2+(L_y-y)^2}}\right]$$

so:
$$\frac{n_1 x}{\sqrt{x^2+y^2}} = \frac{n_2(L_x-x)}{\sqrt{(L_x-x)^2+(L_y-y)^2}}$$

$$\frac{n_1 x}{d_1} = \frac{n_2(L_x-x)}{d_2}$$

$$\therefore \boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

This was a homework problem. Please do the homework, and <u>check</u> the solutions carefully.