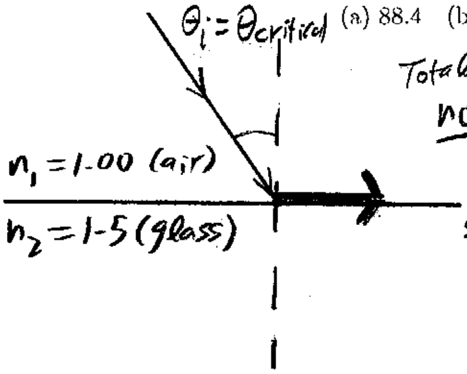
# PHYS4C Fall 2023 Mid-Term Exam 1 SOLUTIONS

(1) What is the critical angle (in degrees) for visible light going from air (with $n = 1.00$) into glass (with $n = 1.5$)?




(a) 88.4 (b) 78.6 (c) 41.8 (d) 18.3 (e) The critical angle is not defined.

Total internal reflection occurs when $n_1 > n_2$, not when $n_1 < n_2$, so $\boxed{\theta_{critical} = \text{undefined}}$.

Formally, $n_1 \sin\theta_{critical} \geq n_2 \cancel{\sin(90°)} = 1$

so: $\theta_{critical} \geq \arcsin(n_2/n_1) = \arcsin(1.5/1.00)$

$$\Rightarrow \boxed{\begin{array}{l} \theta_{critical} = \text{undefined} \\ \text{choice (e)} \end{array}}$$

(2) How much electromagnetic energy is contained in one cubic meter near the Earth's surface if the intensity of sunlight under clear skies is 1000 W/m$^2$

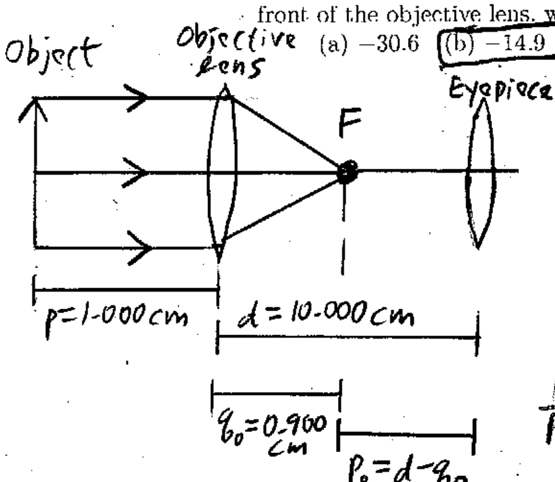
(a) $3.3 \times 10^{-6}$ J (b) 3.3 J (c) 0.003 J (d) $10^{-4}$ J (e) $3.0 \times 10^5$ J

$I = c\, u_{av}$

$$u_{av} = \frac{I}{c} = \frac{(1000\ \frac{W}{m^2})}{(3.00 \times 10^8\ \frac{m}{s})} = 3.3 \times 10^{-6}\ \frac{W}{m^2} \cdot \frac{s}{m} = \boxed{\begin{array}{l} 3.3 \times 10^{-6}\ \frac{J}{m^3} = u_{av} \\ \text{choice (a)} \end{array}}$$

(3) A compound microscope is made with an objective lens ($f_o = 0.900$ cm) and an eyepiece ($f_e = 1.072$ cm). The lenses are separated by a distance of 10.000 cm. If an object is 1.000 cm in front of the objective lens, where will the final image of the eyepiece be located (in cm)?

(a) −30.6 (b) −14.9 (c) −23.3 (d) −11.0 (e) −9.0




Find $-q_e$, the image distance for the 2nd lens, the eyepiece.

For any multi-lens system, the first lens (here, $q_o$) has an image distance that equals the object distance of the second lens (here, $p_e$), so: $p_e = d - q_o$, and $d = 10.000$ cm.

For the first lens (the objective lens):

$$\frac{1}{p_o} + \frac{1}{q_o} = \frac{1}{f_o}, \text{ so } \frac{1}{1.000\text{ cm}} + \frac{1}{q_o} = \frac{1}{+0.900\text{ cm}},$$

so: $q_o = +9.000$ cm.

For the second lens (the eyepiece):

$p_e = d - q_o = 10.000\text{ cm} - 9.000\text{ cm} = +1.000$ cm

$$\frac{1}{p_e} + \frac{1}{q_e} = \frac{1}{f_e}, \text{ so } \frac{1}{+1.000\text{ cm}} + \frac{1}{q_e} = \frac{1}{1.072\text{ cm}}$$

$$\Rightarrow \boxed{\begin{array}{l} q_e = -14.9\text{ cm} \\ \text{choice (b)} \end{array}}$$

(4) Bob Dylan began his music career at a 3000 W radio station in Duluth, MN. In other words, the radio transmitter radiated a total power of 3000 W, in all directions. At a distance of 100 km from this transmitter, what is the intensity of the signal, in W/m$^2$? (The area of a sphere is $4\pi r^2$.)

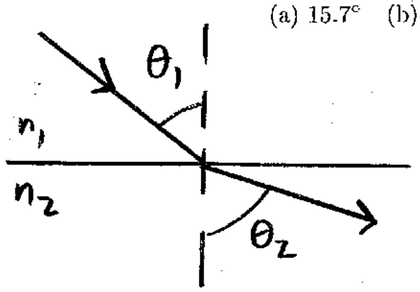
(a) $1.4 \times 10^{-6}$ (b) $2.4 \times 10^{-7}$ (c) $2.4 \times 10^{-8}$ (d) $5.6 \times 10^{-9}$ (e) $8.2 \times 10^{-10}$

$$I = \frac{\text{Power}}{\text{Area}} = \frac{P}{4\pi r^2} = \frac{(3000\,w)}{4\pi (10^5 m)^2} = \boxed{I = 2.4 \times 10^{-8}\ w/m^2}$$

$\Rightarrow$ choice (c)

(5) The laws of refraction and reflection are the same for sound as for light. The speed of sound in air is 340 m/s and in water it is 1510 m/s. If a sound wave in air approaches a plane water surface at an angle of incidence of 12.0°, what is the angle of refraction?

(a) 15.7° (b) 23.5° (c) 27.8° (d) 67.4° 72.7°




$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$n = c/v$

$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$

$\left(\frac{v_2}{v_1}\right)\sin\theta_1 = \sin\theta_2$

$\rightarrow \left(\frac{1510\,m/s}{340\,m/s}\right)\sin(12.0^0) = \sin\theta_2$

$\boxed{\theta_2 = 67.4^0}$

$\Rightarrow$ choice (d)

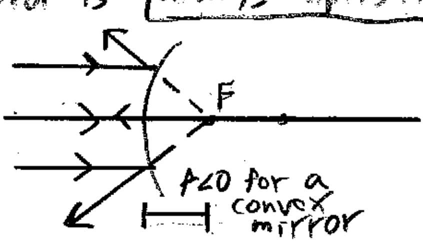
(6) All convex mirrors have radius of curvature $R < 0$. All convex mirrors therefore have focal length $f < 0$. All convex mirrors have image distance $q < 0$. All mirrors have:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f},$$

where $p$ is the object distance and $p > 0$ always, by convention. All convex mirrors therefore show images that are:

(a) Upright, diminished, and real
(b) Upright, diminished, and virtual
(c) Inverted, diminished, and real
(d) Inverted, magnified, and virtual
(e) Inverted, diminished, and virtual

By the sign conventions for mirrors, an upright image has $M > 0$.
$M = -\frac{q}{p} > 0$, since $q < 0$ and $p > 0$.
The image for a convex (or flat) mirror is $\boxed{\text{always upright}}$.




If $p > 0$ and $f < 0$, then $q < 0$ since $(1/p) + (1/q) = (1/f)$.
With $q < 0$, the image for a convex mirror must be $\boxed{\text{always virtual}}$.

A diminished image has $|M| < 1$, which means $0 < M < +1$.
All convex mirrors have $f < 0$, so $\frac{1}{f} < 0$,
so: $\frac{1}{p} + \frac{1}{q} = \frac{1}{f} < 0$,
and: $\frac{1}{p} + \frac{1}{q} < 0$.
This implies $\frac{1}{p} < -\frac{1}{q}$
and so: $\frac{q}{p} < -1$
and: $-\frac{q}{p} < +1$.

$\rightarrow$ Since $M = -q/p$, $M < +1$.
From at left, $M > 0$,
so: $0 < M < +1$
and so: $|M| < 1$.
The image in a convex mirror is therefore $\boxed{\text{always diminished}}$.

Box your final answer. No work = No credit on this part.

**Problems.**

(A) (4 points each) A 0.050-W laser emits a beam with $\lambda = 632.8 \times 10^{-9}$ m and circular cross-section of radius $1.50 \times 10^{-3}$ m. The laser beam travels in empty space. (The area of a circle is $\pi r^2$.)

(i) What is $k$, **in MKS units**? MKS units are the SI units used throughout Phys 4A, 4B, and 4C. MKS units are based on the meter, kilogram, and second. and units defined specifically by them, for example, Newtons (N), Joules (J), Tesla (T), Volts (V), and Coulombs (C). Feet and inches are not MKS units. Centimeters (cm) and kilometers (km) are also not MKS units, even though they are metric units.

(ii) What is the intensity associated with this beam, in W/m$^2$?

(iii) What is the maximum electric field in this beam, **in MKS** units?

(iv) What is the maximum magnetic field in this beam, **in MKS** units?

(v) Suppose a pulse of this laser radiation is emitted that lasts 1 nanosecond = 1 ns = $10^{-9}$ s. How much energy is in this pulse, in Joules?

$P = 0.050\ W = 5.0 \times 10^{-2} W$

$\lambda = 632.8 \times 10^{-9} m = 6.328 \times 10^{-7} m$

$r = 1.50 \times 10^{-3} m$

$A(\text{circle}) = \pi r^2$

(i) $k = \frac{2\pi}{\lambda} = \frac{2\pi}{(6.328 \times 10^{-7} m)} = \boxed{k = 9.929 \times 10^{6} m^{-1}}$

(ii) $I = \frac{P}{A} = \frac{P}{A(\text{circle})} = \frac{(5.0 \times 10^{-2} W)}{\pi (1.50 \times 10^{-3} m)^2} = \boxed{I = 7.1 \times 10^{3} \frac{W}{m^2}}$

(iii) $I = \frac{E_{max} B_{max}}{2\mu_0} = \left(\frac{E_{max}}{2\mu_0}\right)\left(\frac{E_{max}}{c}\right)$ since $\frac{E_{max}}{B_{max}} = c$.

$I = \frac{E_{max}^2}{2\mu_0 c} \Rightarrow E_{max} = \sqrt{2\mu_0 c I}$

$= \sqrt{(2)(4\pi \times 10^{-7} \frac{Tm}{A})(3.00 \times 10^{8} \frac{m}{s})(7.1 \times 10^{3} \frac{W}{m^2})}$

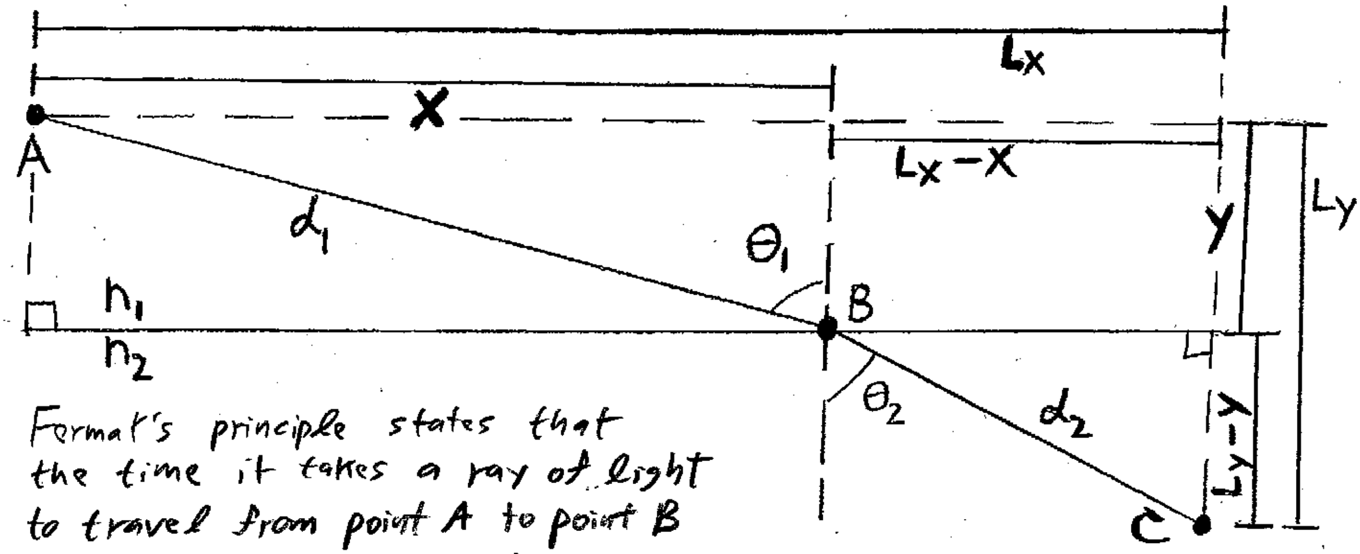
$\boxed{E_{max} = 2.3 \times 10^{3} V/m}$

(iv) $B_{max} = \frac{E_{max}}{c} = \frac{2.3 \times 10^{3} V/m}{3.00 \times 10^{8} m/s} = \boxed{B_{max} = 7.7 \times 10^{-6} T}$.

(v) $\text{Power} = \frac{\text{Energy}}{\text{time}}$, so $E = Pt = (0.050 W)(10^{-9} s)$

$\boxed{E = 5.0 \times 10^{-11} J}$.

(B) A ray of light travels from point A to point B in a medium with index of refraction $n_1$, and then from point B to point C in another medium that has index of refraction $n_2$. Fermat's principle states that this ray of light travels from point A to point B, and then from point B to point C, in the minimum possible time. Use the figure on this page and a little geometry, trigonometry, and calculus to show that this implies Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$. [Hint: recall that time = distance/speed, and that $n = c/v$. Recall also that to find the minimum time it takes a ray of light to travel a distance, take the first derivative with respect to distance, and set it equal to zero.]




Fermat's principle states that the time it takes a ray of light to travel from point A to point B and from point B to point C must be a minimum, so:

$$t_{total} = t_{AB} + t_{BC} = \frac{d_1}{v_1} + \frac{d_2}{v_2} = \frac{n_1 d_1}{c} + \frac{n_2 d_2}{c}, \text{ since } n \equiv c/v.$$

$$t_{total} = \left[\frac{n_1\sqrt{x^2+y^2}}{c} + \frac{n_2\sqrt{(L_x-x)^2+(L_y-y)^2}}{c}\right],$$

by the Pythagorean theorem: $d_1^2 = x^2+y^2$ and: $d_2^2 = (L_x-x)^2 + (L_y-y)^2$.

To find the minimum, remember the trick from calculus:

$$\frac{dt_{total}}{dx} = 0 = \left[\frac{2xn_1}{c\sqrt{x^2+y^2}} - \frac{2(L_x-x)n_2}{c\sqrt{(L_x-x)^2+(L_y-y)^2}}\right]$$

$$\Rightarrow \frac{n_1 x}{\sqrt{x^2+y^2}} = \frac{n_2(L_x-x)}{\sqrt{(L_x-x)^2+(L_y-y)^2}}$$

$$\frac{n_1 x}{d_1} = \frac{n_2(L_x-x)}{d_2}$$

$$\therefore \boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}.$$

Do the homework problems, and the practice exams. They help!