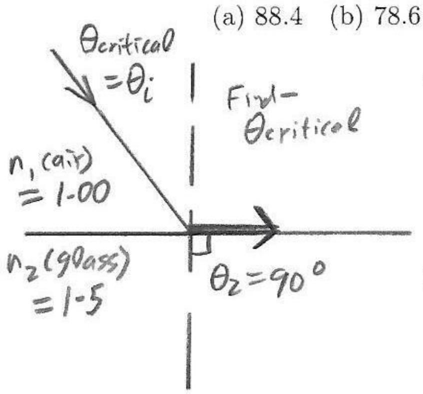
SOLUTIONS to Mid-Term Exam 1, 2025 Spring, Phys 4C

(1) What is the critical angle (in degrees) for visible light going from air (with $n_{\mathrm{air}} = 1.00$) into glass (with $n_{\mathrm{glass}} = 1.5$)?

(a) 88.4 (b) 78.6 (c) 41.8 (d) 18.3 (e) The critical angle is not defined.



Total internal reflection occurs when $n_1 > n_2$, NOT when $n_1 < n_2$ so $\boxed{\theta_{critical} = \text{undefined}}$ $\boxed{\Rightarrow \text{choice (e)}}$

Formally, $n_1 \sin\theta_{critical} \geq n_2 \sin(\underbrace{90°}_{=1})$

so: $\theta_{critical} \geq \arcsin(n_2/n_1) = \arcsin(1.5/1.00) = \arcsin(1.5)$,

so $\boxed{\theta_{critical} \text{ is undefined.}}$

(2) A lens has a convex front surface with a radius of curvature of 20 cm, and a back surface that is flat. It is made of dense flint glass, with $n = 1.67$. What is the focal length (in cm) of this lens?

(a) 20 (b) 30 (c) 40 (d) 10 (e) 50

$R_1 = +20$ cm, positive (>0) since convex.

$R_2 \to \infty$ since flat

Find f

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \underbrace{\frac{1}{R_2}}_{=0}\right]$$

$$= (1.67-1)\left[\frac{1}{+20\text{cm}}\right] = \frac{0.67}{+20\text{cm}}$$

$$f = \frac{+20\text{ cm}}{0.5} = \boxed{f = +30\text{ cm}}$$

$\boxed{\Rightarrow \text{choice (b)}}$

(3) At a distance of 13.5 km from a radio transmitter, the amplitude of electric field strength is measured to be 0.35 V/m. What is the total power emitted by the transmitter? [Hint: The area of a sphere is $4\pi r^2$.]

Find – I

(a) 1.63×10^4 W (b) 4.66×10^4 W (c) 1.31×10^5 W (d) 3.72×10^5 W (e) 8.43×10^6 W

Intensity $I = \frac{\text{Power}}{\text{Area}}$,

so $\text{Power} = IA = I\,4\pi r^2$,

and $r = 13.5\,\text{km} = 1.35 \times 10^4\,\text{m}$.

$$I = \frac{E_{max} B_{max}}{2\mu_0} = \frac{E_{max}^2}{2\mu_0 c},$$

since $\frac{E_{max}}{B_{max}} = c$.

$$\Rightarrow \text{Power} = \left(\frac{E_{max}^2}{2\mu_0 c}\right) 4\pi r^2$$

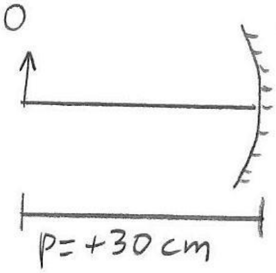
$$\text{Power} = \frac{(0.35\,\text{V/m})^2\, \cancel{4\pi}\, (1.35 \times 10^4\,\text{m})^2}{2\,(\cancel{4\pi} \times 10^{-7}\,\frac{\text{Tm}}{\text{A}})(3.00 \times 10^8\,\frac{\text{m}}{\text{s}})}$$

$\boxed{\text{Power} = 3.72 \times 10^5\,\text{W}}$

$\Rightarrow$ choice (d)

(4) An object is placed 30 cm in front of a concave mirror with a focal length of 15 cm. What is the magnification? (a) 2 (b) 1 (c) $-\frac{1}{2}$ (d) $-\frac{1}{4}$ (e) -1

Find – $M = -\frac{q}{p}$



concave mirror, $f = +15\,\text{cm}$, with $f > 0$ since it's concave.

$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ for all mirrors.

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

$$= \frac{1}{15\,\text{cm}} - \frac{1}{30\,\text{cm}} = \frac{2-1}{30\,\text{cm}} = \frac{1}{30\,\text{cm}}$$

$$q = +30\,\text{cm}$$

$$\text{Magnification } M = -\frac{q}{p} = \frac{-(+30\,\text{cm})}{+30\,\text{cm}}$$

$\boxed{M = -1}$

$\Rightarrow$ choice (e)

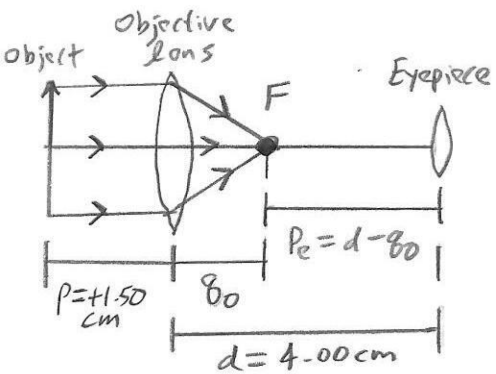
(5) A microscope is made of two lenses. The one in front is called the objective lens, and the one in back is called the eyepiece. The objective lens has $f_O = +1.00$ cm, and the eyepiece has $f_e = +1.25$ cm. The two lenses are separated by a distance of 4.00 cm. If an object is 1.50 cm in front of the objective lens, where (in cm) will the final image from the eyepiece be located?

(a) −5.00 (b) −9.00 (c) −10.00 (d) −12.00 (e) −23.00

Find q_e, the image of the second lens, the eyepiece.

Remember: For any multi-lens system, the first lens (here, q_0) forms an image where the object of the second lens is.

Therefore, $q_0 + p_e = d$ = the distance between the two lenses = 4.00 cm.



For the first lens (the objective),

$$\frac{1}{p_0} + \frac{1}{q_0} = \frac{1}{f_0}, \text{ so } \frac{1}{q_0} = \frac{1}{+1.00\text{cm}} - \frac{1}{+1.50\text{cm}} = \frac{1.50-1}{+1.50\text{cm}} = \frac{0.50}{+1.50\text{cm}},$$

so $q_0 = +3.00$ cm and $p_e = d - q_0 = +4.00\text{cm} - 3.00\text{cm} = +1.00$cm.

For the second lens (the eyepiece),

$$\frac{1}{p_e} + \frac{1}{q_e} = \frac{1}{f_e}, \text{ so } \frac{1}{q_e} = \frac{1}{+1.25\text{cm}} - \frac{1}{+1.00\text{cm}} = \frac{1-1.25}{+1.25\text{cm}} = \frac{-0.25}{+1.25\text{cm}},$$

so $\boxed{q_e = -5.00 \text{ cm}}$

$\Rightarrow$ choice (a)

(6) All convex mirrors have radius of curvature $R < 0$. All convex mirrors therefore have focal length $f < 0$. All convex mirrors have image distance $q < 0$. All mirrors have:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f},$$

where p is the object distance and $p > 0$ always, by convention. All convex mirrors therefore show images that are:

(a) Inverted, diminished, and real

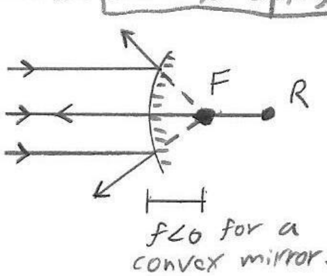
(b) Inverted, magnified, and virtual

(c) Inverted, diminished, and virtual

(d) Upright, diminished, and real

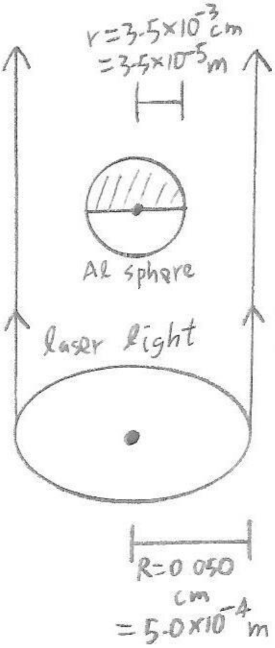
(e) Upright, diminished, and virtual

By the sign conventions for mirrors, an upright image has $M > 0$.
$M = -q/p$, and $q < 0$ and $p > 0$ for a convex mirror, so $M > 0$ for a convex mirror.
The image for a convex (or flat) mirror is therefore $\boxed{\text{always upright}}$.



If $p > 0$ and $f < 0$, always true for a convex mirror, then $q < 0$ always for a convex mirror, since $(1/p) + (1/q) = (1/f)$. With $q < 0$, the image for a convex mirror is $\boxed{\text{always virtual}}$.

A diminished image has $|M| < 1$, which means $0 < M < 1$.
All convex mirrors have $f < 0$, so $1/f < 0$,
so: $\frac{1}{p} + \frac{1}{q} = \frac{1}{f} < 0$, and: $\frac{1}{p} + \frac{1}{q} < 0$.
This implies: $1/p < -1/q$, so $q/p < -1$, and $-q/p < +1$.
Since $M = -q/p$, $M < +1$.
From the figure at left, $M > 0$, so $0 < M < 1$.
Therefore, $|M| < 1$, which means the image in a convex mirror is $\boxed{\text{always diminished}}$.



Box your final answer. No work = No credit on this part.

Problems.

(A) A laser with a circular cross section is used to suspend a spherical aluminum bead in Earth's gravitational field, by balancing the force coming from the radiation pressure against the downward gravitational force. The bead has a mass of 1.0×10^{-6} grams and a radius of 3.5×10^{-3} cm. Some helpful information is:

$g = 9.80665$ m/s^2; $F_{\text{gravity}} = mg$; Volume of a sphere $= \frac{4}{3}\pi r^3$; Area of a circle $= \pi r^2$.

(i) What is the radiation intensity (in W/m^2) needed to support the aluminum bead, assuming the bead reflects all the incident laser light?
(ii) What is the radiation intensity (in W/m^2) needed to support the aluminum bead, assuming the bead absorbs all the incident laser light?
(iii) If the laser beam has a radius of 0.050 cm, what is the power (in Watts) required for this laser?
(iv) What is the maximum magnetic field (in T) of this laser light?
(v) What is the maximum electric field (in V/m) of this laser light?

$F_{gravity} = mg = (1.0\times10^{-6} g)\left(\frac{1 kg}{10^3 g}\right)\left(9.80665 \frac{m}{s^2}\right) = 9.8\times10^{-9}$ kg m/s^2.

This force must balance the force upward from the radiation pressure from the laser.

Since pressure = force/area, $P_{rad} = \frac{F_{gravity}}{A} = \frac{mg}{\pi r^2}$, since the aluminum sphere reflects the light shining on its cross-sectional area, $A = \pi r^2$.

(i) For complete reflection, $P_{rad} = 2I/c$,

so $I_r = \frac{c P_{rad}}{2} = \frac{c}{2}\left(\frac{mg}{\pi r^2}\right) = \frac{(3.00\times10^8 \frac{m}{s})(9.8\times10^{-9} kg\ m/s^2)}{(2\pi)(3.5\times10^{-5} m)^2}$

$\boxed{I_r = 3.8\times10^8 W/m^2}$.

(ii) For complete absorption, $P_{rad} = I/c$, so $I_a = c P_{rad} = \boxed{I_a = 7.6\times10^8 W/m^2} = 2 I_r$

(iii) I = Power/Area, so Power $= I(\pi r^2)$.

For complete reflection, Power $= I_r \pi (5.0\times10^{-4} m)^2 = 300$ W.

For complete absorption, Power $= I_a \pi (5.0\times10^{-4} m)^2 = 600$ W,

so the power required is the greater of these, $\boxed{Power = 600 W}$.

(iv) $I = \frac{E_{max} B_{max}}{2\mu_0} = \frac{c B_{max}^2}{2\mu_0}$, since $E_{max}/B_{max} = c$.

$B_{max} = \sqrt{2\mu_0 I/c} = \sqrt{2(4\pi\times10^{-7} Tm/A)(7.6\times10^8 w/m^2)/(3.00\times10^8 m/s)}$

$\boxed{B_{max} = 2.5\times10^{-3} T}$ for the maximum B_{max}, since $I_a = 2I_r > I_r$.

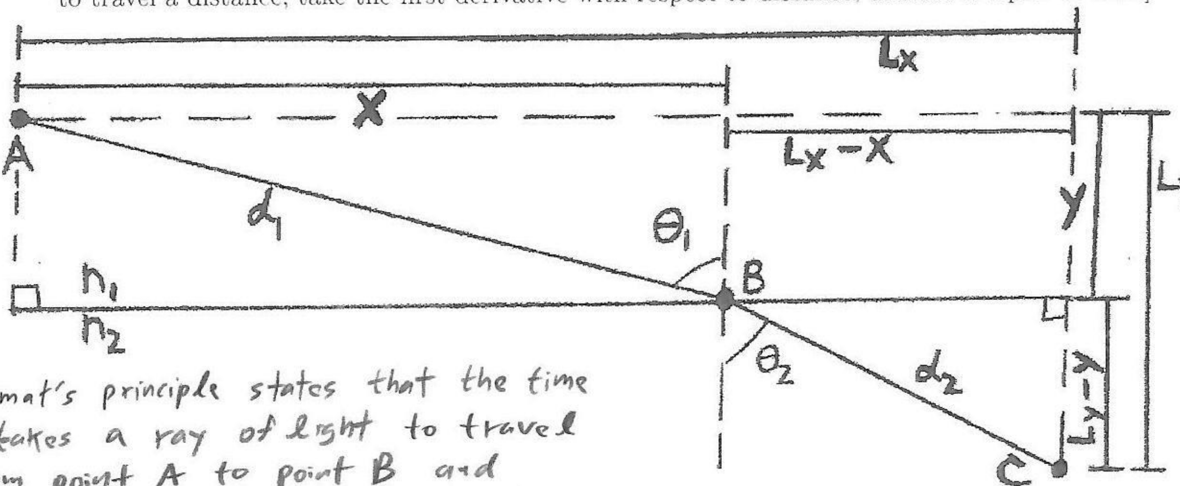
(v) $\frac{E_{max}}{B_{max}} = c$, so $E_{max} = c B_{max} = (3.00\times10^8 \frac{m}{s})(2.5\times10^{-3} T)$

$\boxed{E_{max} = 7.6\times10^5 V/m}$.

Alternatively,

$E_{max} = \sqrt{2\mu_0 c I} = 7.6\times10^5$ V/m. ✓

(B) A ray of light travels from point A to point B in a medium with index of refraction n_1, and then from point B to point C in another medium that has index of refraction n_2. Fermat's principle states that this ray of light travels from point A to point B, and then from point B to point C, in the minimum possible time. Use the figure on this page and some geometry, trigonometry, and calculus to show that this implies Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$. [Hint: recall that time = distance/speed, and that $n = c/v$. Recall also that to find the minimum time it takes a ray of light to travel a distance, take the first derivative with respect to distance, and set it equal to zero.]



Fermat's principle states that the time it takes a ray of light to travel from point A to point B and from point B to point C must be a <u>minimum</u>, so: $t_{total} = t_{AB} + t_{BC} = \frac{d_1}{v_1} + \frac{d_2}{v_2} = \frac{n_1 d_1}{c} + \frac{n_2 d_2}{c}$, since $n \equiv c/v$.

$$t_{total} = \left[\frac{n_1\sqrt{x^2+y^2}}{c} + \frac{n_2\sqrt{(L_x-x)^2+(L_y-y)^2}}{c}\right],$$

since by the Pythagorean theorem, $d_1^2 = x^2 + y^2$
and: $d_2^2 = (L_x - x)^2 + (L_y - y)^2$.

To find the minimum t_{total}, remember the trick from calculus:

$$\frac{d\,t_{total}}{dx} = 0 = \left[\frac{2x n_1}{c\sqrt{x^2+y^2}} - \frac{2(L_x-x)n_2}{c\sqrt{(L_x-x)^2+(L_y-y)^2}}\right]$$

$$\Rightarrow \frac{n_1 x}{\sqrt{x^2+y^2}} = \frac{n_2 (L_x - x)}{\sqrt{(L_x-x)^2+(L_y-y)^2}}$$

$$\frac{n_1 x}{d_1} = \frac{n_2 (L_x - x)}{d_2}$$

$$\therefore \boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

Do the homework problems <u>yourself</u>, and the practice exams. They help!