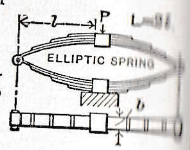
Formulas for Elliptic Springs



S = fiber stress, assumed as 80,000 pounds per square inch for steel;
E = modulus of elasticity = 25,400,000;
P = maximum carrying capacity of spring;
F = deflection of spring under load P; the deflection under any load less than P is directly proportional to the deflection under full load;
n = number of plates or leaves in half-elliptic spring, or one-half total number of leaves in full-elliptic spring;
b = width of plates; h = thickness of plates; $L = 2l$ = effective span;
r = ratio of number of full length leaves to total number of leaves.

Type of Spring	To be Found	Formulas	Examples
Full-elliptic, with all leaves graduated	P	$P = \frac{2\,Snbh^2}{3\,L}$ or, for steel springs: $P = 53{,}000\,nbh^2 \div L$	Steel spring, number of leaves in one-half of spring = 5; Width of plates = 2 inches; Thickness of plates = $\frac{1}{4}$ inch; Effective span, $L = 33$ inches. Find safe load. $P = [53{,}000 \times 5 \times 2 \times (\frac{1}{4})^2] \div 33$ = 1000 pounds, approx.
	F	$F = \frac{SL^2}{2\,Eh}$ or, for steel springs: $F = 0.0016\,L^2 \div h$	
Full-elliptic, with portion of leaves graduated	P	$P = \frac{2\,Snbh^2}{3\,L}$ or, for steel springs: $P = 53{,}000\,nbh^2 \div L$	Same spring as above, with two leaves in each half extending full length of span. Find deflection. Here $r = 2 \div 5 = 0.4$. $F = 0.0031\frac{33^2}{(2+0.4)\frac{1}{4}}$ = 5.625 inches.
	F	$F = \frac{1}{2+r} \times \frac{SL^2}{Eh}$ or, for steel springs: $F = 0.0031\frac{L^2}{(2+r)\,h}$	
Half-elliptic, with all leaves graduated	P	$P = \frac{2\,Snbh^2}{3\,L}$ or, for steel springs: $P = 53{,}000\,nbh^2 \div L$	Same spring as above, but half-elliptic. Find deflection, if all leaves are fully graduated. $F = (0.00079 \times 33^2) \div \frac{1}{4}$ = 3.44 inches.
	F	$F = \frac{SL^2}{4\,Eh}$ or, for steel springs: $F = 0.00079\,L^2 \div h$	
Half-elliptic, with portion of leaves graduated	P	$P = \frac{2\,Snbh^2}{3\,L}$ or, for steel springs: $P = 53{,}000\,nbh^2 \div L$	Same spring as above, half-elliptic, with three leaves extending full length of span. Find deflection. $F = 0.0016\frac{33^2}{(2+0.6)\frac{1}{4}}$ = 2.68 inches.
	F	$F = \frac{1}{2\,(2+r)} \times \frac{SL^2}{Eh}$ or, for steel springs: $F = 0.0016\frac{L^2}{(2+r)\,h}$	

Elliptic Spring Tables. — The "Elliptic Spring Tables" give the maximum static load and the deflection under this load when the length or span of the spring, the number of leaves, and the width and thickness of the leaves are known. The maximum static load as given in these tables induces a fiber stress of 80,000 pounds per square inch in the leaves, and the oscillations may increase this to 100,000 pounds. The successive leaves are supposed to be regularly shortened or "graduated" in the full-elliptic spring. In the half-elliptic spring it is assumed that one-quarter of the whole number of leaves extend to the end of the spring and that the remainder are graduated. In the tables:

L = span, or length of spring in inches, not including band.
F = deflection under load P in inches for both half- and full-elliptic springs;
P = maximum static load in pounds;
N = number of leaves in the half-elliptic spring, or number of leaves in one of the halves of a full-elliptic spring;
B = width of leaves in inches.

As an example of the use of the tables, find the maximum load and deflection of a half-elliptic spring having 5 leaves made of ⅜ by 4-inch steel and having a length of 30 inches. By referring to the tables, the deflection is found to equal 1.47 inch under a maximum static load. This load is found as follows: $N \times B = 5 \times 4 = 20$. Then the maximum static load equals 10 times the value found in the column headed "2," or $10 \times 500 = 5000$ pounds.

The table can also be used for finding the maximum static load and the deflection if all the leaves of the spring extend the full length of the span. The maximum load in this case will be the same as when the leaves are graduated, but the deflection will be less. For full-elliptic springs it will be two-thirds of the amounts given in the column under "F" headed "Full." For half-elliptic springs it will be one-third of the amounts given in the column headed "Full." As an example, find the maximum static load and the deflection of a full-elliptic spring having 7 leaves in each half, 4 inches wide, all extending the full length of the span. The leaves are made of ⅜-inch thick steel and the span is 30 inches. From the table the deflection is found as follows: $3.20 \times \frac{2}{3} = 2.14$ inches. The value of $N \times B = 4 \times 7 = 28$. The maximum load, then, equals:

The value in column "2" × 10 = 5000
The value in column "8" = 2000
Maximum load = 7000 pounds

Elliptic Spring Tables

(For explanation see text above)

Thickness of Steel ⅛ Inch

L Inches	F Inches		Values of P for Varying Values of $N \times B$								
	Half	Full	1	2	3	4	5	6	7	8	9
5	0.12	0.27	167.0	334	501	668	835	1002	1169	1336	1503
6	0.18	0.39	139.0	278	417	556	695	834	973	1112	1251
7	0.24	0.52	119.0	238	357	476	595	714	833	952	1071
8	0.31	0.68	104.0	208	312	416	520	624	728	832	936
9	0.40	0.87	92.5	185	278	370	463	555	648	740	833
10	0.49	1.06	83.3	167	250	333	417	500	583	666	750
11	0.59	1.29	75.8	152	227	303	379	455	531	606	682
12	0.70	1.54	69.5	139	209	278	348	417	487	556	626