Code Design for Noncoherent Detection in Satellite Communication Systems

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***Abstract*—In this paper, we propose a waveform design that is robust to non-coherent detection for quadrature phase-shift keying (QPSK) and quadrature amplitude modulation (QAM) schemes. We propose a complex constellation design waveforms based on the quaternary complex Hadamard matrices. The advantage of the proposed waveforms is that they have the largest noncoherent minimum distance in the complex domain and meet the maximum Hamming distance in the binary domain.**

***Index Terms*—noncoherent detection, rotationally invariant codes, satellite transmission.**

I. Introduction

Noncoherent detection scheme has been applied to many areas of communications. One example is noncoherent receivers for satellite communication in which the fading channels can be modeled by Rayleigh distribution. Video Broadcasting via Satellite 2nd Generation (DVB-S2X) has been the most popular choice for providing a mechanism that adapts to variations of channel conditions. In the specific system scenario on the forward link of a satellite system, a signal processing technique, namely precoding, is implemented to account for the mitigation of interference on to the user signals. . However, acquiring channel state information (CSI) can be challenging due to errors in estimation and meeting the timely fashion for such fast varying channels.

Therefore, noncoherent detection would be a good candidate for such channels [1]. In our study, the noncoherent waveform design is intended for wide range of satellite communication systems in which adaptive coding and modulation (ACM) is employed. In one particular scenario is the noncoherent detection of the physical layer header (PLH) of the DVB-S2X superframe. In such system, the phase of the received symbols are modified by unknown arbitrary value, which remains constant during the transmission of each codeword.

The early design of communication separately focused on modulation systems though focused separately on modulation and correcting codes. However, the solution to the problem of increasing the transmission rate without bandwidth expansion is to use a high-order constellation. Shannon already introduced in [2] the idea of combining coding with nonbinary modulation (i.e., high-order constellations), which is often referred to as coded modulation (CM), to emphasize that not only coding but also the mapping from the code bits to the constellation symbols is essential.

There has been lot of designs for noncoherent channels in the literature [3]–[10]. The main approaches essentially follow two strategies. The first strategy is based on finding the capacity achieving channel codes with an optimized, i.e., optimum distribution, constellation for a given noncoherent channel model [3]–[5]. Note that finding the optimum solution jointly as well as finite constellation sets is an open problem [11]. The second strategy is to design the codebook for a fix underlying constellation shape for noncoherent detection. Main idea is to maximize so called *noncoherent* minimum distance. For the cases of M-ary phase-shift keying (MPSK), amplitude-phase shift keying (APSK), and quadrature amplitude modulation (QAM) constellations have been studied in [6]–[10]. Knopp and Leib in [6] based on *free* module developed module-phase coded for MPSK modulation. Their code design proceeds by excluding the unwanted codewords that results ambiguity when phase shifted. Due to the fact that noncoherent distance is not a true metric, the study in [6] mainly relies on a numerical approach to search for good codes and to determine the minimum noncoherent distance. Sun and Leib in [7] extends these results to construct and analyze good codes for noncoherent detection by the use of an analytical approach. They achieve that by demonstrating the relation between the noncoherent distance and those of Euclidean and Lee distances. The study of block-coded modulation for MPSK, QAM and APSK constellations are presented in the following works [8]–[10]. Kayhan and Montorsi in [12] proposed a simply search method for generating a binary short-length rate-compatible families of codes that are robust to noncoherent detection for MPSK constellations. Their method is based on greedy algorithm to construct a family of rotationally invariant (RI) codes with respect to MPSK modulation. The resultant code after eliminating the repetition codewords is suitable for noncoherent detection. The minimum distance achieved for those codes are compared with the optimum maximum minimum distance for existing linear binary codes. The greedy type algorithm can be computationally very complex and not be globally optimal. It will be desirable to search design solutions in existing structures (e.g., Hadamard codes).

In this work, we will be describing the noncoherent waveform design for precoded PLH headers in DVB-S2X. The PLH headers in DVB-S2X and DVB-T have constant lengths, however, it has been shown in [12] that variable length coding technique for PLH results in performance gain. Our objective therefore is to design short finite-block length waveforms in complex constellation such as quadrature phase shift keying (QPSK) for PLH that are robust to noncoherent detection. We show that the proposed codes achieve the maximum noncoherent distance in the complex domain. We demonstrate a simple mapping of those complex codewords to equivalent binary linear codes. The minimum distance of the resultant binary codewords meet the bounds of maximum minimum distance for a given codeword and message lengths.

The rest of the paper is organized as follows. In Section II, we discuss transmission and assumptions made, followed by the complex code set construction in Section III. Mapping the complex waveform to an equivalent binary code sets are presented in Section IV. A few conclusions are drawn in Section VI.

The following notations are used in this paper. All boldface lower case letters indicate column vectors and upper case letters indicate matrices, $()^T$ denotes transpose operation, and $|.|$ is the scalar magnitude.

II. System model

In the present work, under the realm of noncoherent detection, the system is considered to be without carrier phase tracking with the flat fading channel where the phase rotation is considered to be independent of the amplitude variation of channel. Mathematically, we can formulate the system model as

$$y_k \quad = \quad s_k e^{j\theta_k} + n_k, \tag{1}$$

where s_k is the transmitted symbol, n_k is the additive white Gaussian noise (AWGN) with variance, σ^2, j denotes complex number, and θ_k is the phase change of the symbol undergoing through the channel for $1 \leq k \leq N$ and N is the codeword length. The θ_k is normally modeled as a random process with first order statistic uniform in $[0, 2\pi]$. With the above discussion and underlying problem, it will be reasonable to assume that phase θ_k stays constant over the codeword length, N. In other words, we assume that the coherence time is much greater than the codewords length,

$$y_k \quad = \quad s_k e^{j\theta} + n_k. \tag{2}$$

III. Construction of noncoherent detection codes

The objective of our problem is to design waveforms of length N that are robust for the noncoherent detection specifically in QPSK transmission scheme. Let us for a moment look at our original problem from the viewpoint of coding in MPSK transmission as discussed in [6]. First, we consider the transmission of codewords of length N from a codebook $\mathscr{C}$ that are vectors in the ring of integers modular M, namely, $\mathbb{Z}_M$-module[1]. A codeword can then be expressed in the following form,

$$\mathbf{c}_i = [c_{i,0}, c_{i,1}, \ldots, c_{i,N-1}]^T, \tag{3}$$

[1] A module over a ring R is an Abelian group M, which can be considered as a generalization of the notion of vector space over a field.

where $c_{i,j} \in \mathbb{Z}_M$ for $0 \leq j \leq N-1$, $0 \leq i \leq |\mathscr{C}| - 1$ and $|\cdot|$ here denotes the cardinality of code $\mathscr{C}$. We can show that there exists an isomorphism $\varphi : \mathscr{C} \mapsto \mathscr{S}$ from an Abelian group $(\mathscr{C}, \oplus)$ to $(\mathscr{S}, \otimes)$, where $\oplus$ is symbol-by-symbol modular M addition, $\mathscr{S} = \{\mathbf{s} = f(\mathbf{c}) \mid \mathbf{c} \in \mathscr{C}\}$, and $\otimes$ is symbol-by-symbol multiplication. The mapping between $\mathbb{Z}_M$-module and the transmitted MPSK signal vectors in the complex plane can be written as

$$\begin{aligned} \mathbf{s}_i &= f(\mathbf{c}_i) && (4)\\ &= [F_{MPSK}(c_{i,0}), \ldots, F_{MPSK}(c_{i,N-1})]^T, && (5) \end{aligned}$$

where $F_{MPSK}(\cdot)$ can be expressed as,

$$F_{MPSK}(c_{i,j}) = \exp\left[j\frac{2\pi}{M}c_{i,j}\right], \tag{6}$$

where $c_{i,j} \in \mathbb{Z}_M$ for $0 \leq j \leq N-1$ and $0 \leq i \leq |\mathscr{C}| - 1$. From received symbol expression (1), we can formulate the received complex codeword as

$$\mathbf{y} = \mathbf{s}_i e^{j\theta} + \mathbf{n}, \tag{7}$$

where $\mathbf{y} = [y_0, y_1, \ldots, y_{N-1}]^T$, $\mathbf{s} = [s_{i,0}, s_{i,1}, \ldots, s_{i,N-1}]^T$, and $\mathbf{n} = [n_0, n_1, \ldots, n_{N-1}]^T$. It can be shown that the maximum-likelihood (ML) decoder for this scheme is given by

$$\widehat{\mathbf{s}} = \underset{\mathbf{s}_i \in \mathscr{S}}{\operatorname{argmin}} |\mathbf{s}_i^H \mathbf{y}|. \tag{8}$$

The pairwise error probability depends on so-called noncoherent distance between two arbitrary codewords $\mathbf{c}_i$ and $\mathbf{c}_j$ and is given by

$$d_{NC}(i,j) = N(1 - |\rho_{i,j}|), \tag{9}$$

where $\rho_{i,j}$ is the normalized inner product between the i-th and j-th codewords given by

$$\begin{aligned} \rho_{i,j} &= \frac{1}{N}\mathbf{s}_i^H \mathbf{s}_j && (10)\\ &= \frac{1}{N}\sum_{k}^{N} \exp\left[j\frac{2\pi}{M}(c_{i,k} - c_{j,k})\right]. && (11) \end{aligned}$$

If we assume the transmission is over a constant arbitrary phase shift θ (e.g., for QPSK case $\theta \in \{0, \pi/2, \pi, 3/2\pi\}$) then there should not be any two codewords that are rotated versions of each other and corresponds to the same input sequence, namely, are not RI. In other words, $\mathbf{c}_k e^{j\theta} \neq \mathbf{c}_m$, for $\theta \in \{\pi/2, \pi, 3/2\pi\}$ and all k and m. The rotation in this context is constraints to QPSK constellations, however this should not limit us to apply for other constellations. In case of RI, it will lead to a catastrophic behaviour, since rotated codewords cannot be distinguished on the receiver side. Our objective is to design complex codewords $\mathbf{s}_i$ for $1 \leq i \leq |\mathscr{S}| - 1$ such that they have large noncoherent minimum distance, d_{NC} and are not RI. The optimal d_{NC} can be achieved if the codewords are orthogonal with the value of $d_{NC} = N$. The codewords are non-RI if the codebook $\mathscr{C}$ does not contain any of codewords

$$\mathbf{c}_i = [\alpha, \alpha, \ldots, \alpha]^T, \tag{12}$$

where $\alpha \in \mathbb{Z}_M$. Let us now denote the QPSK/QAM constellations with Gray mapping in 2-bit binary and the corresponding complex symbol as

$$\{00, 10, 11, 01\} \leftarrow \{e^{j\theta_0}, e^{j\theta_1}, e^{j\theta_2}, e^{j\theta_3}\} \tag{13}$$

where $\theta_0 = 0$, $\theta_1 = \pi/2$, $\theta_2 = \pi$, and $\theta_3 = 3/2\pi$. Note that the proposed construction should not be any different under any arbitrary phase shift, ϕ, (e.g., $\phi = \pi/4$ results π-QPSK constellation with $\theta_0 = \pi/4$, $\theta_1 = 3/4\pi$, $\theta_2 = 5/4\pi$, and $\theta_3 = 7/4\pi$). If we denote $b_0 = 00$, $b_1 = 10$, $b_2 = 11$ and $b_3 = 01$ we can show that it forms a noncyclic group under modular 4 addition, which is isomorphic to non-cyclic Klein four-group $(\mathbb{Z}_2 \times \mathbb{Z}_2, \times)$ shown in the following table,

$+$	b_0	b_1	b_2	b_3
b_0	b_0	b_1	b_2	b_3
b_1	b_1	b_0	b_3	b_2
b_2	b_2	b_3	b_0	b_1
b_3	b_3	b_2	b_1	b_0

Denoting $\beta_0 = e^{j\theta_0}$, $\beta_1 = e^{j\theta_1}$, $\beta_2 = e^{j\theta_2}$ and $\beta_3 = e^{j\theta_3}$ forms a group under multiplication operator is isomorphic to additive Abelian group $(\mathbb{Z}_4, +)$ as shown in the following table,

$\times$	β_0	β_1	β_2	β_3
β_0	β_0	β_1	β_2	β_3
β_1	β_1	β_2	β_3	β_0
β_2	β_2	β_3	β_0	β_1
β_3	β_3	β_0	β_1	β_2

Since Klien four-group is not isomorphic to additive Abelian group, $(\mathbb{Z}_2 \times \mathbb{Z}_2, \times) \cong (\mathbb{Z}_4, +)$ RI complex codewords cannot be mapped to binary RI codewords and vice versa. Therefore, because of this limitation we will not be attempting to solve our problem by designing a good binary codes that corresponds to complex codewords having large minimum noncooherent distance and are not RI. Instead, we will be solving the problem in the complex waveform domain. To satisfy the minimum noncoherent distance, our code design is based on the complex quaternary Hadamard matrix namely the Elliot-Rao Hadamard recursive matrix construction method [13]. Let

$$\mathbf{C}_1 = \begin{bmatrix} \beta_0 & \beta_3 \\ \beta_0 & \beta_1 \end{bmatrix}, \mathbf{C}_2 = \begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_1 \\ \mathbf{C}_1 & -\mathbf{C}_1 \end{bmatrix}. \tag{14}$$

Then for $m \geq 3$, an Elliot-Rao Hadamard matrices represented by the recursion as follows,

$$\mathbf{C}_m = \begin{bmatrix} \mathbf{C}_{m-1} & \mathbf{C}_{m-1} \\ \mathbf{C}_1 \otimes \mathbf{H}_{m-2} & -\mathbf{C}_1 \otimes \mathbf{H}_{m-2} \end{bmatrix}, \tag{15}$$

where $\mathbf{H}_m$ is a Sylvester-Hadamard matrix of order 2^m. We recall that the Sylvester-Hadamard matrix of order 2 is $\mathbf{H}_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ and of order 2^{p+1} for $p = 1, 2, ...$ is $\mathbf{H}_{2^{p+1}} = \begin{bmatrix} \mathbf{H}_{2^p} & \mathbf{H}_{2^p} \\ \mathbf{H}_{2^p} & -\mathbf{H}_{2^p} \end{bmatrix}$ [14]. Then, for any $p = 1, 2, ...$, $\mathbf{H}_{2^p}\mathbf{H}_{2^p} = 2^p \mathbf{I}_{2^p \times 2^p}$, where $\mathbf{I}_{N \times N}$ is the $N \times N$ identity matrix. As mentioned earlier, the orthogonality of $\mathbf{C}_m$ is preserved by any phase rotation, ϕ, of codewords. An example of complex Hadamard matrix for $m = 2$ is given by

$$\mathbf{C}_2 = [\mathbf{c}_0, \mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3] = \begin{bmatrix} \beta_0 & \beta_0 & \beta_0 & \beta_0 \\ \beta_0 & \beta_2 & \beta_0 & \beta_2 \\ \beta_0 & \beta_3 & \beta_2 & \beta_1 \\ \beta_0 & \beta_1 & \beta_2 & \beta_3 \end{bmatrix}, \tag{16}$$

where $\mathbf{c}_0 = [\beta_0, \beta_0, \beta_0, \beta_0]^T$, $\mathbf{c}_1 = [\beta_0, \beta_2, \beta_3, \beta_1]^T$, $\mathbf{c}_2 = [\beta_0, \beta_0, \beta_2, \beta_2]^T$ and $\mathbf{c}_3 = [\beta_0, \beta_2, \beta_1, \beta_3]^T$. Unlike the case of MPSK complex codewords, $\mathbf{C}_2$ is not a complex codebook simply because there exist a pair of codewords in $\mathbf{C}_2$ and their symbol-by-symbol multiplication result in a vector with all ones. In other words, equivalently there exist a pair of codewords in $\mathbb{Z}_M$-module that their modular M addition results in zero vector. Let columns, $\mathbf{c}_k$, be labeled as α_k for $0 \leq k \leq 4$ then there is a group formed with the α_k elements under the symbol-by-symbol multiplication defined as,

$$\mathbf{c}_k \odot \mathbf{c}_j = [c_{0,k}c_{0,j}, c_{1,k}c_{1,j}, \ldots, c_{3,k}c_{3,j}]^T. \tag{17}$$

Therefore, α_k's form a group under the elementwise $\odot$ multiplication, which is shown in Table III below.

$\odot$	α_0	α_1	α_2	α_3
α_0	α_0	α_1	α_2	α_3
α_1	α_1	α_2	α_3	α_0
α_2	α_2	α_3	α_0	α_1
α_3	α_3	α_0	α_1	α_2

From the Cayley tables it is obvious that this group is isomorphic to a cyclic group of order 4 and additive Abelian group $(\mathbb{Z}_4, +)$. Note that $(\mathbb{Z}_4, +)$ is not isomorphic to the non-cyclic group $(\mathbb{Z}_2 \times \mathbb{Z}_2, \times)$. Due to this reason it is not considered to be linear codes, as there exists linear dependency in $\mathbf{C}_2$, specifically, $\alpha_1 \odot \alpha_3 = \alpha_0$.

Even though the proposed complex Hadamard matrices, $\mathbf{C}_m$, do not belong to a complex codewords as in MPSK case, they achieve the maximum noncoherent distance due to orthogonality and they are not RI by nature of their constructions. Designing complex codes with different lengths and cardinality can be approached by appending complex columns and rows to our proposed complex codes. With this approach it might not be possible to obtain orthogonal codewords for all lengths and size of cardinality however, they at least need to satisfy Welch lower bound (WBE) [15]. In the next section, we will analyse the mapping between the proposed complex codewords to binary codewords.

IV. Binary counterpart codes

As discussed, the proposed complex matrix $\mathbf{C}_m$ having orthogonal columns do not belong to a linear complex codes. However, opposite results are found when mapping the complex codewords in $\mathbf{C}_m$ to a binary counterpart codes. We use the Gray mapping as in (13). To illustrate an example, we take the $\mathbf{C}_2$ and map it to binary codeword with the of (8×4) as

$$\mathbf{C}_2^b = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}. \tag{18}$$

The first two rows are redundant, which can be eliminated to result in a codebook of size (6×4). Selecting the second and third independent columns to construct the generating matrix $\mathbf{G}_2$ as,

$$\mathbf{G}_2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}, \tag{19}$$

which maps the message $\mathbf{x} \in \{0,1\}^{2\times 1}$ to $\mathbf{G}_2\mathbf{x}$. Due to orthogonality property it can be proved that Hadamard codes have minimum distance equal to 2^m. With our proposed construction we have created Hadamard codes with a $[2^{(m+1)} - 2, m, 2^m]$. The minimum distance of Hadamard codes therefore meet the maximum bounds of minimum distance of linear binary codes in [16] is shown in the Table I.

TABLE I
CODE LENGTHS FOR A GIVEN K AND d_{min}

	Binary	Proposed	Codes in [12]
K/d_{min}	4	4	4
2	6	6	6
K/d_{min}	8	8	10
3	14	14	18
K/d_{min}	16	16	20
4	30	30	38
K/d_{min}	32	32	30
5	62	62	60
K/d_{min}	64	64	50
6	126	126	104

V. SIMULATION

In this section, we evaluate the performance of the proposed complex waveforms for the lengths of $N = 8, 32, 64$. In our simulations, the noncoherent transmission is modeled by (2) and ML detection by (8). Whereas the coherent transmission is given by

$$y_k = s_k + n_k. \tag{20}$$

We construct $\mathbf{C}_3$, $\mathbf{C}_5$ and $\mathbf{C}_8$ complex codebooks as described in (15). The elements of each codewords are QAM modulated, $s_k \in \{\beta_0, \beta_1, \beta_2, \beta_3\}$. The selection of a transmitted codeword, $\mathbf{s}_i$, is a mapping from the binary codeword that is generated by $\log_2(N)$ bits and generating matrices, $\mathbf{G}_3$, $\mathbf{G}_5$, $\mathbf{G}_8$. The ML detector for the coherent case can be shown to be expressed as

$$\widehat{\mathbf{s}} = \underset{\mathbf{s}_i \in \mathscr{S}}{\operatorname{argmin}} \Re\{\mathbf{s}_i^H \mathbf{y}\}, \tag{21}$$

where $\Re\{\}$ denotes the real part of the complex number. In Figs. 1 and 2, the bit-error-rate (BER) and frame-error-rate (FER) are performed over the complex AWGN channel.

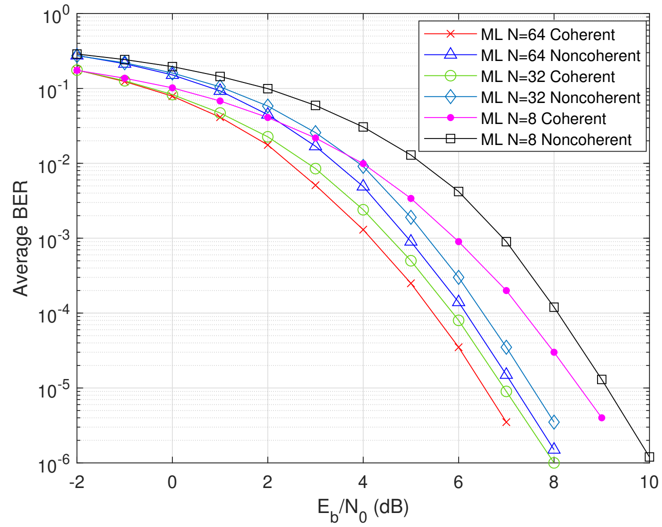

Fig. 1. QAM modulation over the complex AWGN.

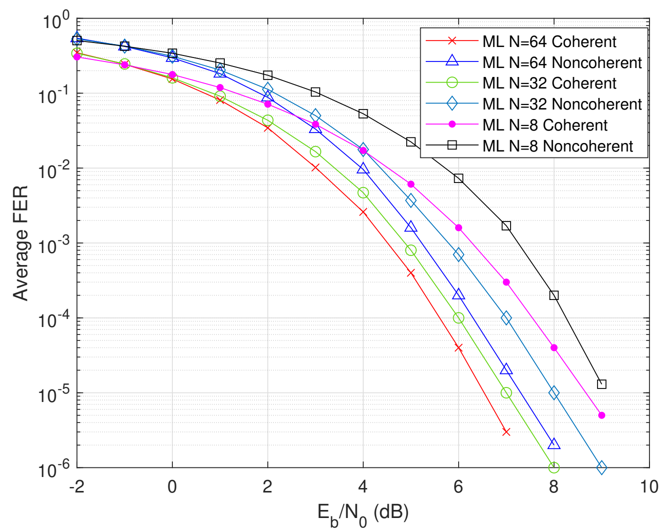

Fig. 2. QAM modulation over the complex AWGN.

As expected, coherent system outperforms the noncoherent system by around 1 dB for all codeword lengths, $N = 8, 32, 64$. We also evaluate BER and FER performances over the real as opposed to complex AWGN channel in Figs. 3 and 4. For high values of E_b/N_0 (e.g., $8 - 9$ dB) BER and FER performance of the noncoherent and coherent system gets very close to each other.

VI. Conclusion

In this paper, we have introduced a waveform design that are robust to noncoherent detection for quadrature phase shift keying (QPSK) and quadrature amplitude modulation (QAM) modulation. We design the waveform in the complex constellation, which is based on the quaternary complex Hadamard matrices. Unlike other greedy construction methods, which is based on the building a generator matrix for binary codewords, our method is based on complex Hadamard matrices without any computation complexity. The resulting complex waveforms are orthogonal, meet the maximum noncoherent distance as well as maximum hamming distance in binary domain.

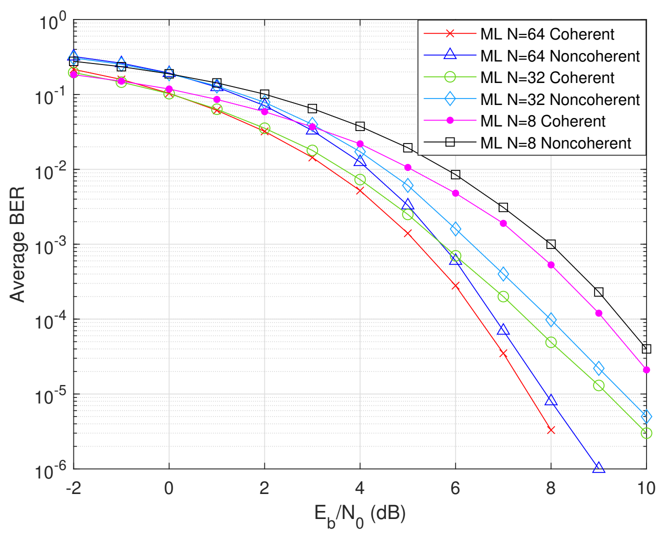

Fig. 3. QAM modulation over the real AWGN.

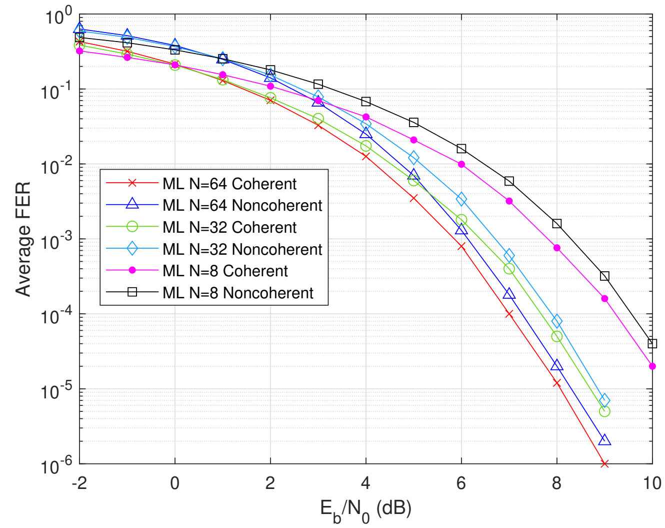

Fig. 4. QAM modulation over the real AWGN.

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