27 Cards Trick: What's Your Favorite Number?

<u>What the audience sees.</u>

You have a stack of cards, and tell the audience that there are 27 cards in it. You ask a volunteer from the audience to choose any card from the stack, memorize it, and show it to the audience. Then the volunteer puts the card back into the stack. Next, you ask the volunteer to pick his/her favorite number from 0 to 26 (inclusive). You explain that you will put all the cards into 3 stacks 3 times, and each time the volunteer will have to tell you which of the 3 stacks contains his/her card (but not tell you which card it is). You also say that after putting all the cards back together, you will hand the stack to the volunteer. He/she should count off his/her chosen number of cards, and the next card will be the one he/she chose. Then you do as explained, and indeed after the volunteer counts off his/her chosen number of cards, the next card is the one the audience saw earlier.

<u>How you do it.</u>

The position of the card in the end will be determined by how you reassemble the 3 stacks each time. To determine how to reassemble the stacks, convert the volunteer's chosen number to base 3. If it is a one- or two-digit number in base 3, write zero(s) in front to make it a three-digit number.

In the beginning you get the stack with the volunteer's card in it, and you do not know its value or position. Keep the cards face down in one hand and start putting them into 3 stacks, face up: you put one card into the first stack, one into the second, one into the third, then one into the first one again, and so on. When you are done (there will be 9 cards in each stack), the volunteer tells you which stack contains his/her card. Recall the units (last) digit of the volunteer's number in base 3. Reassemble the three stacks so that the card with the volunteer's card goes to the position according to the table below.

Digit	Position of the stack with the volunteer's card
2	Top
1	Middle
0	Bottom

Then you turn the whole stack over, and repeat the process of putting the cards into 3 stacks face up. Again, you are told which stack contains the volunteer's card. Put this stack in the position determined by the table according to the second digit of the base 3 representation of the volunteer's number. You turn the whole stack over again, and put the cards into 3 stacks one last time. This time the stack with the volunteer's card has to go in the position according to the first digit of the base 3 representation of his/her number. Now, you turn the whole stack over again, and hand the stack to the volunteer. You tell him/her to count off his/her chosen number of cards from the top and show the next card after that.

Why it works.

Let's trace the cards in the stacks as you reassemble them 3 times. First, let's see where the cards from the top, middle, or bottom stack when you the reassemble them the first time end up. Namely, let's mark the cards that are put at the top in the first reassemble red, the middle ones yellow, and the bottom ones green. When you turn the stack over, red and green cards will switch positions:

(face up)   →   (face down)   (1)
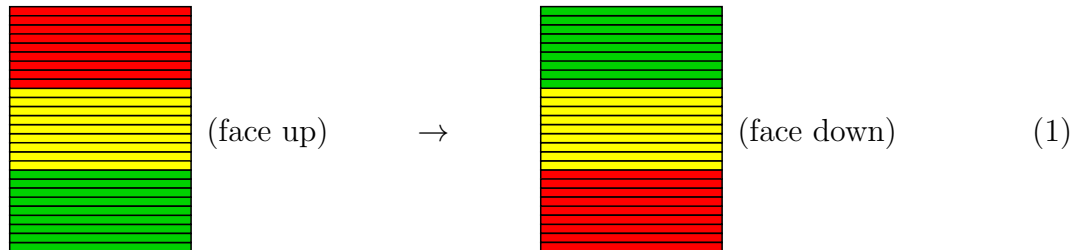
When you put them into 3 stacks again, they take the following positions:

After reassembling the cards a second time and turning the whole stack over, they are as follows:

(face up)   →   (face down)   (2)
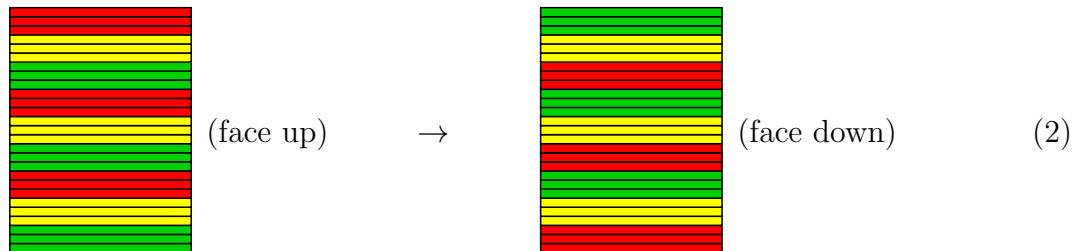
When you put the cards into 3 stacks again, the stacks are:

Finally, after reassembling the cards a third time and turning the whole stack over, you get:

(face up)   →   (face down)   (3)
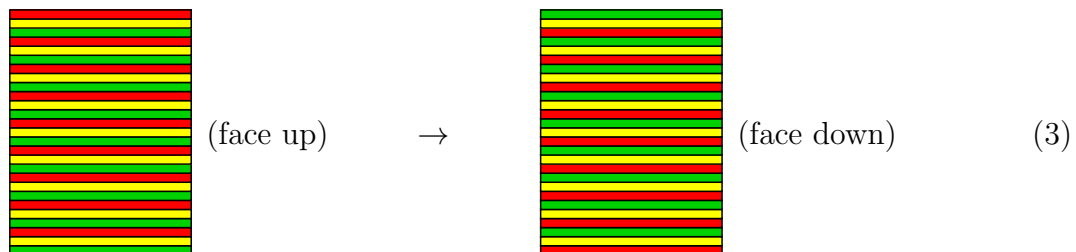
We see that the number of cards above any green card is a multiple of 3 (which corresponds to the last digit being 0 in the base 3 representation), the number of cards above any yellow card is 1 larger than a multiple of 3 (corresponding to the last digit being 1), and the number of cards above any red card is 2 larger than a multiple of 3 (corresponding to the last digit being 2).

Next, let's see what happens to the cards put at the top, middle, or bottom the second time your reassemble them. If we again mark them red, yellow, and green correspondingly, their final position will be as in (2) above. Notice that the number of cards

above any green card is either a multiple of 9, or 1 more than a multiple of 9, or 2 more than a multiple of 9. These numbers have 0 for the second digit in base 3 representation. The number of cards above any yellow card is 3, 4, or 5 more than a multiple of 9 (corresponding to the second digit being 1 in the base 3 representation). The number of cards above any red card is 6, 7, or 8 more than a multiple of 9 (corresponding to the second digit being 2 in the base 3 representation).

Finally, the positions of the cards put at the top, middle, or bottom in the third reassemble have final positions as in (1) above. Namely, all the green (bottom) cards have less than 9 cards above them (corresponding to the first digit being 0 in the base 3 representation), all the yellow (middle) cards have from 9 to 17 cards above them (corresponding to the first digit being 1), and all the red (top) cards have from 18 to 26 cards above them (corresponding to the first digit being 2).

Thus, we see that the position of the stack containing the volunteer's card the first time you reassemble the stacks determines the last digit in the base 3 representation of the number of cards above the volunteer's card in the end, the position of the stack the second time you reassemble them determines the second digit, and the position of the stack the third time your reassemble them determines the third digit.

Example.

Suppose the volunteer chose the number 7. You write it in base 3:

$$7 = 2 \times 3 + 1 = 21_3 = 021_3.$$

According to the table above, you should put the stack with the volunteer's card in the middle the first time you reassemble the stacks, on top the second time, and on bottom the third time.

So, after you reassemble the cards the first time, you put the stack with the volunteer's card in the middle, and the card will be one of the 10th through 18th cards. It will remain in this middle position when you turn the whole stack over. Then, when you put the cards into three stacks again, it will be one of the 4th through 6th cards in one of the stacks. You put that stack on top, and the card becomes one of the 4th through 6th cards from the top in the whole stack. When you turn the stack over, it will be one of the 4th through 6th cards from the bottom. Then you put the cards into three stacks again, and the volunteer's card will be the 2nd from the top, or 8th from the bottom, in its stack. When you reassemble the stacks a third time, the stack with the volunteer's card goes on bottom, and the card becomes 8th from the bottom of the whole stack. Finally, you turn the whole stack over, and the card becomes 8th from the top. Thus, there are 7 cards on top of it.