Guess 1 Out Of 5 Cards Trick

<u>What the audience sees.</u>

It takes two people to do this trick. We will call them Alice and Bob. Alice has a deck of 52 regular playing cards and explains to the audience that a volunteer will pick any 5 cards from the deck, give them to her, and then she will decide which one of these 5 cards Bob will have to guess. Then she gives the whole deck to a volunteer and asks him/her to pick any 5 cards. The volunteer gives these 5 cards to Alice and keeps the rest. Alice looks at them, and puts 4 of them on the table. Bob can see these 4 cards but does not see the fifth one which is still in Alice's hand (it is the only card she is holding). Bob names the card. Alice shows it to the audience. Bob guessed correctly.

<u>How you do it.</u>

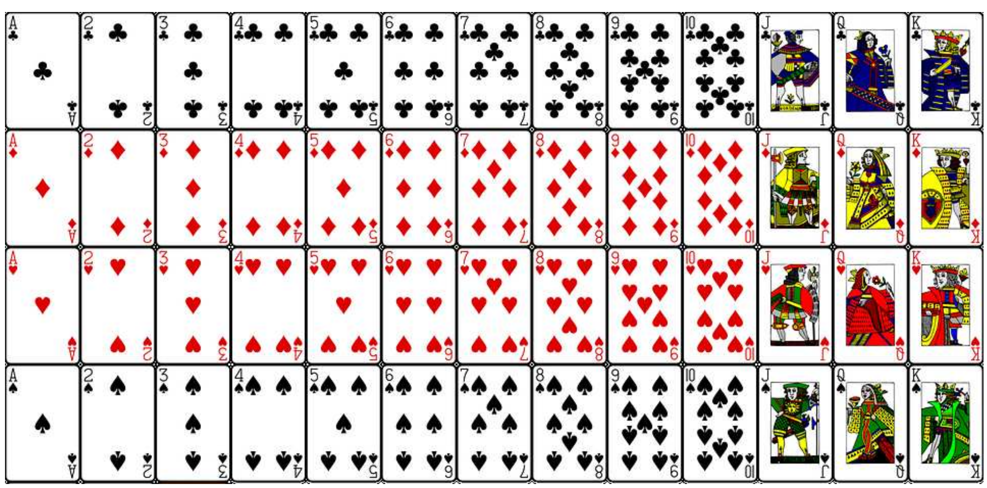
There is no guessing involved. The 4 cards that Alice puts on the table encode information about the fifth one.

There are 4 suits: clubs, diamonds, hearts, spades. Among the 5 cards that Alice gets from the volunteer, at least one suit will repeat. Alice chooses two cards of the same suit. One of them Bob will be "guessing" and the other one Alice puts on the table first. So as soon as Bob sees the suit of the first card, he knows the suit of the one he has to "guess". The remaining 3 cards will be needed to tell Bob the value of the card he is "guessing".

There are six ways to arrange 3 objects in a row:

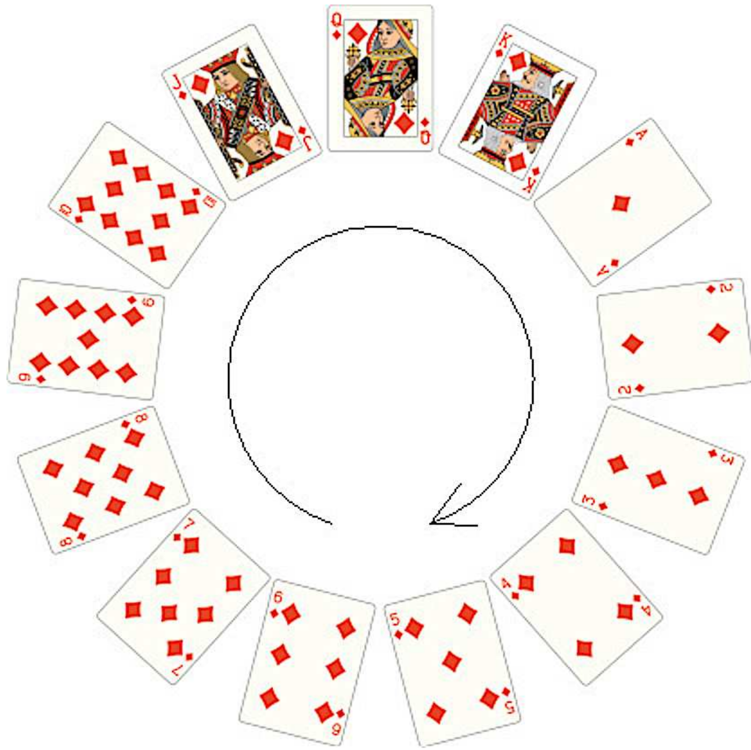
1. ABC	3. BAC	5. CAB
2. ACB	4. BCA	6. CBA

To make use of these orders, however, we need to know which card is A, which one is B, and which one is C. So Alice and Bob agree on the following order of the cards.



Notice that all clubs are first, then all diamonds, then all hearts, then all spades (to help you remember this order, notice that the words clubs, diamonds, hearts, spades are listed in alphabetical order). Within every suit, the cards are arranged in the increasing order of their value: Ace=1, then 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack=11, Queen=12, King=13.

So, first Alice finds two cards of the same suit (if there is more than one pair of cards of the same suit, any pair can be chosen). She looks at their values and finds them in the following circle:



Following the arrow, she finds the "distance" from each card to the other one, and among these two possible distances she chooses the smaller one. It is always less than 7. If "distance" from card 1 to card 2 is smaller than "distance" from card 2 to card 1, Alice will put card 1 on the table first and keep card 2 for Bob to "guess". Otherwise, she will put card 2 on the table first and keep card 1 for Bob to "guess". The "distance" (1, 2, 3, 4, 5, or 6) will need to be encoded in the arrangement of the other three cards. So, she finds the locations of the other three cards in the above order and labels the one that appears first as A, the second as B, and the third as C. Then, on the table, she puts them in the order ABC if she needs to communicate the distance 1, ACB if the distance is 2, and so on (see the list of all six arrangements on the previous page).

Bob looks at the first card and knows the suit. Then he looks at the next three cards and determines their order (ABC, or ACB, etc.) Knowing the order of those 3 cards, he finds the distance (1, or 2, etc.) from the first card to the one he has to "guess".

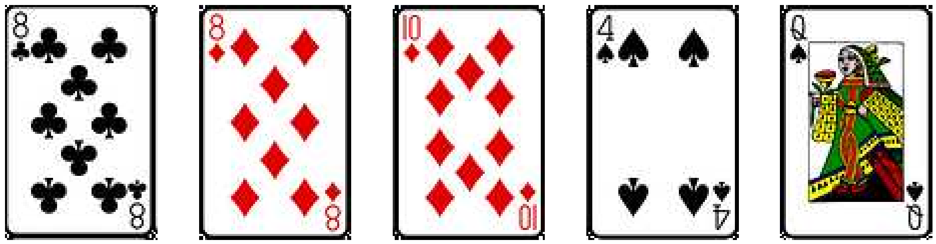
Why it works.

In general, if there are n types of objects and more than n objects, than at least two of the objects are of the same type. In this case, there are 4 suits ("types" of cards), so out of 5 cards at least two must be of the same suit. In mathematics, this idea is called the **Pigeonhole Principle**, because if pigeons fly into pigeonholes and there are more pigeons than holes, then at least two pigeons must be in the same hole.

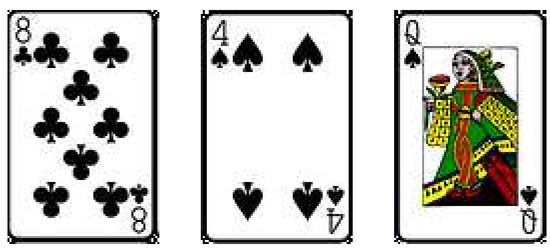
There are 13 cards of each suit. Therefore, for any two cards, the sum of the "distance" (in the direction of the arrow) from one card to the other and the "distance" from the second to the first is 13. The smaller of these two distances is, therefore, at most 6.

Example.

Suppose Alice was given the following cards:

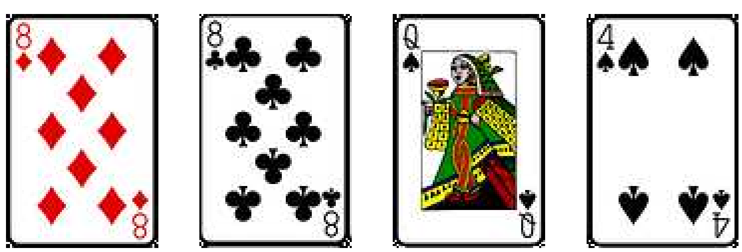


Her first step is to find two cards of the same suit. In this case, she can choose either 8 of diamonds and 10 of diamonds, or 4 of spades and Queen of spades. Let's say she chooses 8 of diamonds and 10 of diamonds. The distance from 8 to 10 is 2 (and the distance from 10 to 8 is 11), so she will put 8 of diamonds first, and will keep Queen of spades for Bob to "guess". She needs to encode the distance 2 using the remaining three cards. First she orders them (clubs first, spades later; between the two spades 4 comes before the Queen), and assigns letters A, B, C:



A B C

To encode the distance 2, she puts them in the order A, C, B. So, she puts all four cards in the following order:

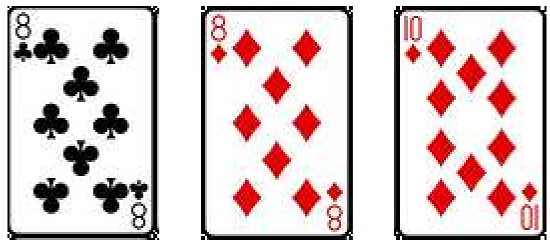


diamonds $\underbrace{\text{A} \qquad \text{C} \qquad \text{B}}_{\text{ACB means distance=2}}$

Bob looks at these cards, sees 8 of diamonds first and immediately knows that the suit is diamonds. The he looks at the next three cards, sees that they are in the order A, C, B, and so he knows the distance is 2. He adds $8 + 2 = 10$ and says "10 of diamonds".

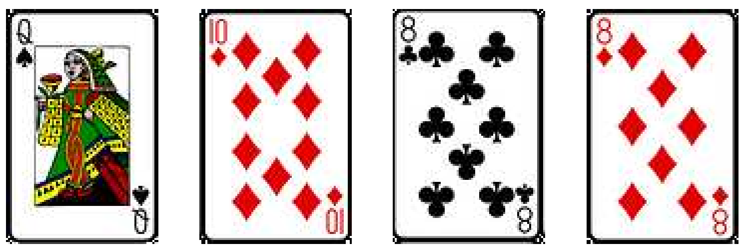
Note. Alice could have chosen 4 of spades and Q of spades as the two cards of the same suit. The rest of the calculation for this choice is shown on the next page. As we will see, the choice given above is easier to work with.

Let's see what would happen if Alice chose 4 of spades and Q of spades as the two cards of the same suit. The distance from 4 to Queen (which has a value of 12) is 8, and the distance from Queen to 4 is 5. So she would have to put the Queen first and keep the 4 for Bob to "guess". She would order the remaining three cards:



A B C

To encode the distance 5, she would put them in the order C, A, B. So, the 4 cards she would put on the table are as follows:



spades $\underbrace{\text{C} \qquad \text{A} \qquad \text{B}}_{\text{CAB means distance=5}}$

Bob would see Queen of spades first and would know the suit was spades. Then he would see that the remaining three cards are in the order C, A, B, and would know the distance was 5. He counts to the 5th card after Queen: King, Ace, 2, 3, 4. So he would say "4 of spades".

Tip: When you have a choice, try to find an "easier" pair. In this example, it was easier to compute 8+2=10 than Q+5=4, so the choice given on the previous page is a better one for Alice to use.