Math 111, Fall 2014 - Homework # 11

Due Monday, December 1, 2014, by 3:00 p.m.

You must show all of your work and explain all of your answers to receive full credit.

1. Define a relation R on $\mathbb{Z}$ by $x\,R\,y$ if $x \cdot y \geq 0$. Prove or disprove the following:
   (a) R is reflexive;
   (b) R is symmetric;
   (c) R is transitive.

   Solution:

2. Let $A = \{1, 2, 3, 4\}$. Give an example of a relation on A that is:
   (a) reflexive and symmetric, but not transitive;
   (b) symmetric and transitive, but not reflexive;
   (c) symmetric, but neither transitive nor reflexive.

   Solution:

3. Let R be an equivalence relation on $A = \{a, b, c, d, e, f, g\}$ such that $a\,R\,c$, $c\,R\,d$, $d\,R\,g$, and $b\,R\,f$. If there are three distinct equivalence classes that result from R, then determine these equivalence classes and determine all elements of R.

   Solution:

4. Define a relation R on $\mathbb{Z}$ as $x\,R\,y$ if and only if $x^2 + y^2$ is even. Prove R is an equivalence relation and determine its distinct equivalence classes.

   Solution:

5. Prove or disprove. If R and S are two equivalence relations on a set A, then $R \cap S$ is also an equivalence relation on A.

   Solution:

6. Describe the partition of $\mathbb{Z}$ resulting from the equivalence relation $\equiv \pmod 3$.

   Solution:

7. Write the addition and multiplication tables for $\mathbb{Z}_8$.

   Solution:

8. Prove or disprove. If $[a], [b] \in \mathbb{Z}_6$ and $[a][b] = [0]$, then either $[a] = [0]$ or $[b] = [0]$.

   Solution: