Math 111, Fall 2014 - Homework # 6

Due Thursday, October 16, 2014, by 4:30 p.m.

Remember that you are required to fully explain all of your solutions.

1. Use the binomial theorem to show that

$$\sum_{k=0}^{n} \binom{n}{k} 6^k = 7^n.$$

Solution:

2. This problem involves lists made from the letters T, H, E, O, R, Y, with repetition allowed.

   (a) How many 4-letter lists are there that do not begin with a T or do not end in a Y?

   (b) How many 4-letter lists are there in which the sequence of letters T, H, E appear consecutively?

   Solution:

3. Prove that if x is an odd integer, then x^3 is odd.

   Solution:

4. Suppose $a, b, c \in \mathbb{Z}$. Prove that if $a|b$ and $a|c$, then $a|(b+c)$.

   Solution:

5. Prove that if $x \in \mathbb{R}$ and $0 < x < 4$, then $\dfrac{4}{x(4-x)} \geq 1$.

   Solution:

6. Prove that if $n \in \mathbb{Z}$, then $n^2 - 3n + 9$ is odd.

   Solution:

7. Prove for every nonnegative integer n that $2^n + 6^n$ is an even integer.

   Solution:

8. Prove that for every two distinct integers a and b, either $\dfrac{a+b}{2} > a$ or $\dfrac{a+b}{2} > b$.

   Solution:

9. Evaluate the proof of the following proposition.

   Proposition. If x and y are integers, then xy^2 has the same parity as x.

   Proof. Assume, without loss of generality, that x is even. Then $x = 2a$ for some integer a. Thus,

   $$xy^2 = (2a)y^2 = 2(ay^2).$$

   Since ay^2 is an integer, xy^2 is even. □

   Solution: