Math 111, Fall 2014 - Homework # 7

Due Thursday, October 23, 2014, by 4:30 p.m.

Remember that you are required to fully explain all of your solutions.

Prove the following statements using direct proof, contrapositive proof, or proof by contradiction.

1. Suppose $x \in \mathbb{R}$. If $x^3 - x > 0$, then $x > -1$.

   Solution:

2. The product of an irrational number and a nonzero rational number is irrational.

   Solution:

3. If $a \equiv b \pmod{n}$, then $\gcd(a, n) = \gcd(b, n)$.

   Solution:

4. Suppose $a \in \mathbb{Z}$. If a^2 is not divisible by 4, then a is odd.

   Solution:

5. If $a \in \mathbb{Z}$ and $a \equiv 1 \pmod{5}$, then $a^2 \equiv 1 \pmod{5}$.

   Solution:

6. If a and b are positive real numbers, then $a + b \geq 2\sqrt{ab}$.

   Solution:

7. Let $a \in \mathbb{Z}$. If $(a+1)^2 - 1$ is even, then a is even.

   Solution:

8. Let $a, b \in \mathbb{Z}$. If $a \geq 2$, then either $a \nmid b$ or $a \nmid (b+1)$.

   Solution:

9. Evaluate the proof of the following proposition.

Proposition. Let $n \in \mathbb{Z}$. If $3n-8$ is odd, then n is odd.

Proof. Assume that n is odd. Then $n = 2k+1$ for some integer k. Then

$$3n - 8 = 3(2k+1) - 8 = 6k + 3 - 8 = 6k - 5 = 2(3k-3) + 1.$$

Since $3k-3$ is an integer, $3n-8$ is odd. $\square$

Solution: