Homework # 1 Solutions

Math 111, Fall 2014
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1. Write the following sets by listing their elements within braces.

   (a) $A = \{x \in \mathbb{R} : x^2 - x = 0\}$
   (b) $B = \{n \in \mathbb{Z} : n^2 < 7\}$
   (c) $C = \{x \in \mathbb{R} : x^2 + 1 = 0\}$
   (d) $D = \{3n + 1 : n \in \mathbb{Z}\}$

   Solution:

   (a) $A = \{0, 1\}$
   (b) $B = \{-2, -1, 0, 1, 2\}$
   (c) $C = \{\}$
   (d) $D = \{\dots, -5, -2, 1, 4, 7, \dots\}$

2. Write each of the following sets in the form $\{x \in S : p(x)\}$ or $\{p(x) : x \in S\}$, where $p(x)$ is a property concerning x and S is the set containing x.

   (a) $A = \{1, 2, 3, 4, \dots, 9\}$
   (b) $B = \{\dots, -8, -4, 0, 4, 8, \dots\}$
   (c) $C = \{1, 8, 27, 64, \dots\}$

   Solution:

   (a) $A = \{n \in \mathbb{N} : n < 10\}$
   (b) $B = \{4n : n \in \mathbb{Z}\}$
   (c) $C = \{n^3 : n \in \mathbb{N}\}$

3. Give an example of three sets $A, B,$ and C such that $A \in B$ and $A \subseteq C$, or state why such an example cannot exist.

   Solution: For example, if $A = \{1\}$, $B = \{\{1\}, 2\}$, and $C = \{0, 1\}$. We can see that $\{1\} \in \{\{1\}, 2\}$ and $\{1\} \subseteq \{0, 1\}$.

4. Find $\mathcal{P}(A)$ and $|\mathcal{P}(A)|$ for $A = \{0, \{1\}, \{1, 2\}, \{\varnothing\}\}$.

Solution:

$$\mathcal{P}(A) = \{\varnothing, \{0\}, \{\{1\}\}, \{\{1,2\}\}, \{\{\varnothing\}\}, \{0, \{1\}\}, \{0, \{1,2\}\}, \{0, \{\varnothing\}\}, \{\{1\}, \{1,2\}\}, \{\{1\}, \{\varnothing\}\},$$
$$\{\{1,2\}, \{\varnothing\}\}, \{0, \{1\}, \{1,2\}\}, \{0, \{1\}, \{\varnothing\}\}, \{0, \{1,2\}, \{\varnothing\}\}, \{\{1\}, \{1,2\}, \{\varnothing\}\},$$
$$\{0, \{1\}, \{1,2\}, \{\varnothing\}\}\}.$$

We can see that $|\mathcal{P}(A)| = 16$, as expected (since $|A| = 4$).

5. True or False: If $\{1\} \in \mathcal{P}(A)$, then $1 \in A$ but $\{1\} \notin A$.

Solution: False. Here is a counterexample. Let $A = \{\{1\}, 1\}$. Then, $\mathcal{P}(A) = \{\varnothing, 1, \{1\}, \{\{1\}, 1\}$. Then, $\{1\} \in \mathcal{P}(A)$, $1 \in A$, and $\{1\} \in A$.

6. True or False: If a set B has one more element than a set A, then $\mathcal{P}(B)$ has at least two more elements than $\mathcal{P}(A)$.

Solution: Since A is arbitrary, the statement is False. Why? Suppose that $A = \varnothing$ and $B = \{1\}$. Then $|A| = 0$ and $|B| = 1$, but $|\mathcal{P}(A)| = 2^0 = 1$ and $|\mathcal{P}(B) = 2^1 = 2$.

7. For the sets $A = \{1, \{1\}\}$ and $B = \{0, 1, 2\}$, write down all of the elements of $A \times B$. What is $|A \times B|$?

Solution: $A \times B = \{(1, 0), (1, 1), (1, 2), (\{1\}, 0), (\{1\}, 1), (\{1\}, 2)$. We can see that $|A \times B| = 6$, as expected since $|A| = 2$, $|B| = 3$, and $|A \times B| = |A| \cdot |B| = 2 \cdot 3 = 6$.

8. For the set $A = \{1, 2\}$ and $B = \{\varnothing\}$, write down all of the elements of $A \times B$ and $\mathcal{P}(A) \times \mathcal{P}(B)$.

Solution: $A \times B = \{(1, \varnothing), (2, \varnothing)\}$.

Since $\mathcal{P}(A) = \{\varnothing, \{1\}, \{2\}, \{1, 2\}\}$ and $\mathcal{P}(B) = \{\varnothing, \{\varnothing\}\}$,

$$\mathcal{P}(A) \times \mathcal{P}(B) = \{(\varnothing, \varnothing), (\varnothing, \{\varnothing\}), (\{1\}, \varnothing), (\{1\}, \{\varnothing\}), (\{2\}, \varnothing), (\{2\}, \{\varnothing\})\}.$$

9. Describe the graph of the ellipse $4x^2 + 9y^2 = 36$ as a subset of $\mathbb{R} \times \mathbb{R}$.
Note: What I'm looking for here is something like:

The ellipse $4x^2 + 9y^2 = 36$ is the set

$$\{(x, y) \in \mathbb{R} \times \mathbb{R} : \underline{\hspace{2cm}}\}$$

(Now you fill in the blank.)

Solution: The ellipse $4x^2 + 9y^2 = 36$ is the set

$$\big\{(x, y) \in \mathbb{R} \times \mathbb{R} : 4x^2 + 9y^2 = 36\big\}$$