Homework # 2 Solutions

Math 111, Fall 2014
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1. Suppose that $A = \{5, 3, 7, 1, 2\}$, $B = \{1, 4, 9\}$, and $C = \{2, 4, 6\}$. Find

   (a) $A \cup B$

   (b) $A \cap C$

   (c) $A - B$

   (d) $B - C$

   (e) $A \cup (B \cap C)$

   Solution:

   (a) $A \cup B = \{1, 2, 3, 4, 5, 7, 9\}$

   (b) $A \cap C = \{2\}$

   (c) $A - B = \{2, 3, 5, 7\}$

   (d) $B - C = \{1, 9\}$

   (e) $B \cap C = \{4\}$, so $A \cup (B \cap C) = \{1, 2, 3, 4, 5, 7\}$

2. Suppose that $A = \{a, b, c\}$ and $B = \{c, d\}$. Find

   (a) $(A \times B) \cap (B \times B)$

   (b) $(A \times B) - (A \times A)$

   (c) $\mathcal{P}(A) \cap \mathcal{P}(B)$

   (d) $\mathcal{P}(B \times B)$

   (e) $\mathcal{P}(A) - \mathcal{P}(B)$

   Solution:

   (a) $A \times B = \{(a, c), (b, c), (c, c), (a, d), (b, d), (c, d)\}$ and
   $B \times B = \{(c, c), (c, d), (d, c), (d, d)\}$. Therefore, $(A \times B) \cap (B \times B) = \{(c, c), (c, d)\}$

   (b) $A \times B = \{(a, c), (b, c), (c, c), (a, d), (b, d), (c, d)\}$ and
   $A \times A = \{(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c)\}$.
   Therefore, $(A \times B) - (A \times A) = \{(a, d), (b, d), (c, d)\}$.

   (c) $\mathcal{P}(A) = \{\varnothing, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{P}(B) = \{\varnothing, \{c\}, \{d\}, \{c, d\}\}$.
   Therefore, $\mathcal{P}(A) \cap \mathcal{P}(B) = \{\varnothing, \{c\}\}$.

(d) $B \times B = \{(c, c), (c, d), (d, c), (d, d)\}$, so

$$\mathcal{P}(B \times B) = \{\varnothing, \{(c, c)\}, \{(c, d)\}, \{(d, c)\}, \{(d, d)\}, \{(c, c), (c, d)\}, \{(c, c), (d, c)\},$$
$$\{(c, c), (d, d)\}, \{(c, d), (d, c)\}, \{(c, d), (d, d)\}, \{(d, c), (d, d)\}, \{(c, c), (c, d), (d, c)\},$$
$$\{(c, c), (c, d), (d, d)\}, \{(c, d), (d, c), (d, d)\}, \{(c, c), (c, d), (d, c), (d, d)\}\}.$$

(e) $\mathcal{P}(A) = \{\varnothing, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{P}(B) = \{\varnothing, \{c\}, \{d\}, \{c, d\}\}$.
Therefore, $\mathcal{P}(A) - \mathcal{P}(B) = \{\{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$.

3. Let $A = \{0, 2, 4, 6, 8\}$ and $B = \{1, 3, 5, 7\}$ have universal set $U = \{0, 1, 2, \ldots, 8\}$. Find

   (a) $\overline{A}$

   (b) $\overline{B}$

   (c) $\overline{A} \times B$

   (d) $\overline{A \cup B}$

   (e) $A - \overline{A}$

   Solution:

   (a) $\overline{A} = \{1, 3, 5, 7\} = B$

   (b) $\overline{B} = \{0, 2, 4, 6, 8\} = A$

   (c) $\overline{A} \times B = B \times B =$

$$\{(1, 1), (1, 3), (1, 5), (1, 7), (3, 1), (3, 3), (3, 5), (3, 7), (5, 1), (5, 3), (5, 5), (5, 7),$$
$$(7, 1), (7, 3), (7, 5), (7, 7)\}.$$

   (d) $A \cup B = \{0, 1, 2, \ldots, 8\} = U$. Therefore, $\overline{A \cup B} = \varnothing$.

   (e) $A - \overline{A} = A - B = \{0, 2, 4, 6, 8\} = A$.

4. Suppose that sets A and B are in a universal set U. Draw Venn diagrams for each of the following:

   (a) $\overline{A \cap B}$

   (b) $\overline{A} \cap \overline{B}$

   (c) $\overline{A \cup B}$

   (d) $\overline{A} \cup \overline{B}$

   Based on these sketches, make a conjecture about the equality of these sets.

   Solution:

   From the sketches in Figures 1-4, we may conjecture that $\overline{A \cap B} = \overline{A} \cup \overline{B}$ and $\overline{A \cup B} = \overline{A} \cap \overline{B}$.
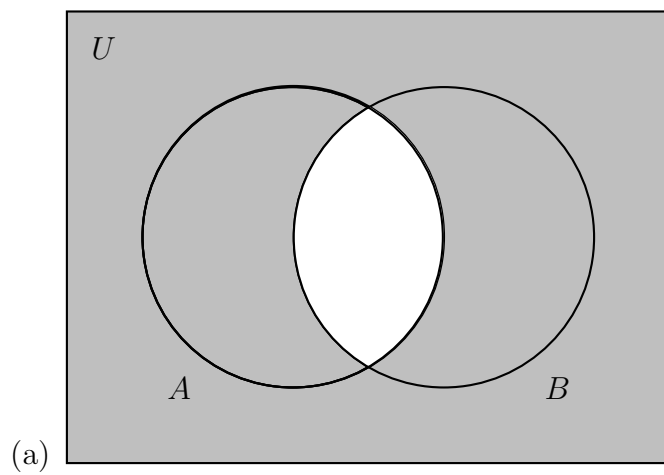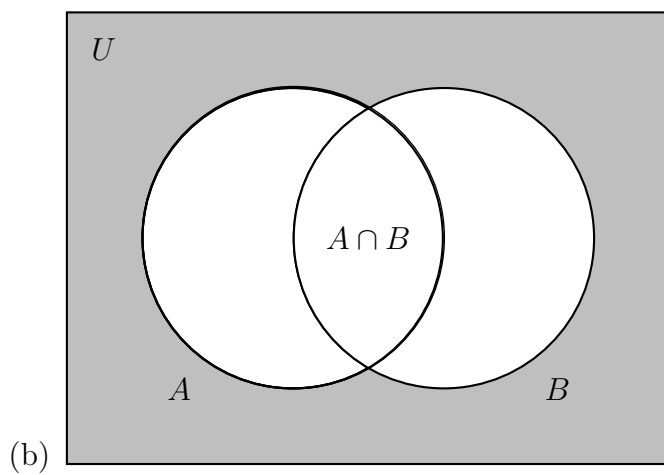
(a)

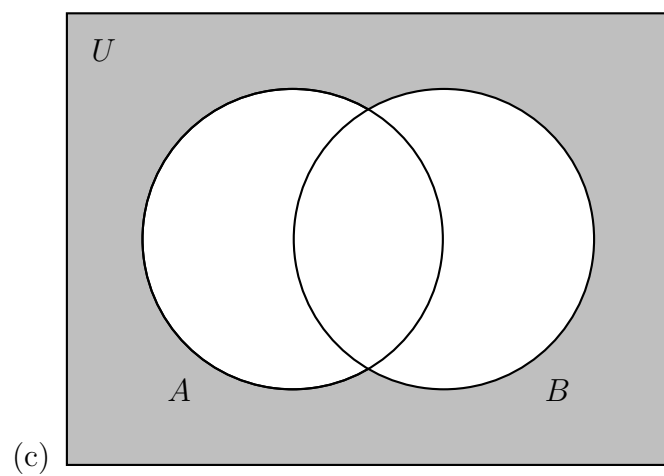Figure 1: $\overline{A \cap B}$

(b)

Figure 2: $\overline{A} \cap \overline{B}$
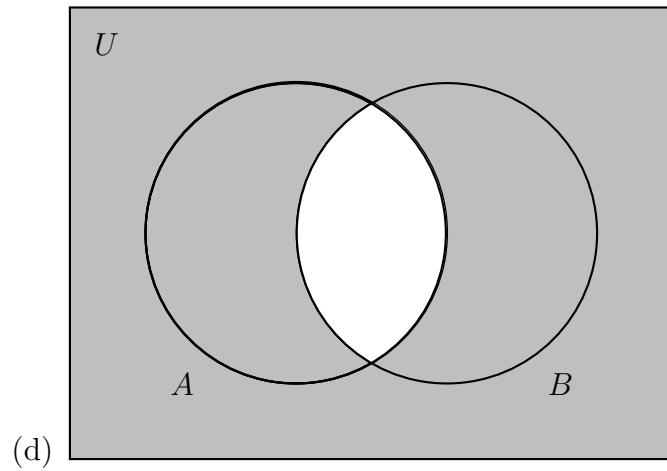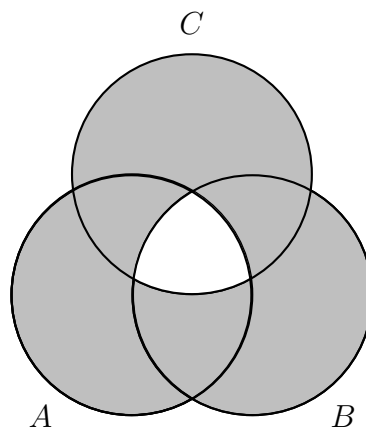

(c)

Figure 3: $\overline{A \cup B}$

(d)

Figure 4: $\overline{A} \cup \overline{B}$

5. Determine the expression involving sets $A, B,$ and C that is illustrated by the Venn diagram below.

Solution: We know from our class notes (and the text) that the Venn diagram of $A \cap B \cap C$ has the region that is white in this Venn diagram shaded and the shaded region white. This tells us that the Venn diagram illustrates $(A \cup B \cup C) - (A \cap B \cap C)$.