Drug Dosage

An Example of a Discrete Model

CURM Background Material, Fall 2014

A doctor prescribes a patient take a pill containing 250 mg of a certain drug ever 4 hours. Assume that the drug is immediately ingested into the bloodstream once taken. Also, assume that every 4 hours, the patient's body eliminates 30% of the drug in his/her bloodstream. Suppose that the patient had 0 mg of the drug in his bloodstream prior to taking the first pill. How much of the drug will be in his bloodstream after 72 hours?

Step 1: Identify the Problem

Determine the relationship between the amount of drug in the bloodstream and time.

Step 2: Identify Relevant Facts about the Problem

• The drug is administered every 4 hours.
• The patient had no drug in his system prioer to taking the initial pill.

Step 3: Choose the Type of Modeling Method

We will use a deterministic discrete model.

Step 4: Make Simplifying Assumptions

Assumptions:
• This system can be modeled by a discrete dynamical system.
• The patient is of normal size and health.
• There are no other drugs being taken that will affect the presscribed drug.
• There are no internal or external factors that will affect drug absorption rate.
• The patient always takes the prescribed dose at the correct time.

Variables: $a(n)$ = amount of drug in the bloodstream after period n (in mg)
n = number of 4-hour periods of time ($n = 0, 1, 2, \ldots$)

Step 5: Construct the Model

$a(n+1)$ = amount of drug in the bloodstream in the future (in mg)
$a(n)$ = amount of drug in the bloodstream currently (in mg)

Define Change as follows:

Change = dose - loss in the system
$\Rightarrow$Change = 250 - 0.30$a(n)$

So, Future = Present + Change:
$a(n+1) = a(n) - 0.30a(n) + 250$
$\Rightarrow a(n+1) = 0.70a(n) + 250,\ a(0) = 0$

Step 6: Solve and Interpret the Model

Solve by hand to obtain: $a(n) = \frac{2500}{3} - \frac{2500}{3} \cdot (0.7)^n$. See class notes for details.

To solve in Maple, use the *rsolve* command.

$drug := rsolve(\{a(n+1) = 0.70 \cdot a(n) + 250, a(0) = 0\}, a(n))$

$$-\frac{2500}{3}\left(\frac{7}{10}\right)^n + \frac{2500}{3} \quad \textbf{(6.1)}$$

Find the value after 72 hours:

$d := unapply(drug, n)$

$$n \rightarrow -\frac{2500}{3}\left(\frac{7}{10}\right)^n + \frac{2500}{3} \quad \textbf{(6.2)}$$

$endtime := \frac{72}{4}$

$$18 \quad \textbf{(6.3)}$$

$result := d(endtime)$

$$\frac{332790528800696517}{400000000000000} \quad \textbf{(6.4)}$$

$evalf(result)$

$$831.9763220 \quad \textbf{(6.5)}$$

Let's plot the values of d(n) and see what type of curve we obtain.

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with(plots) :
pointplot({seq([i, d(i)], i = 0 ..24)}, symbol = solidcircle)
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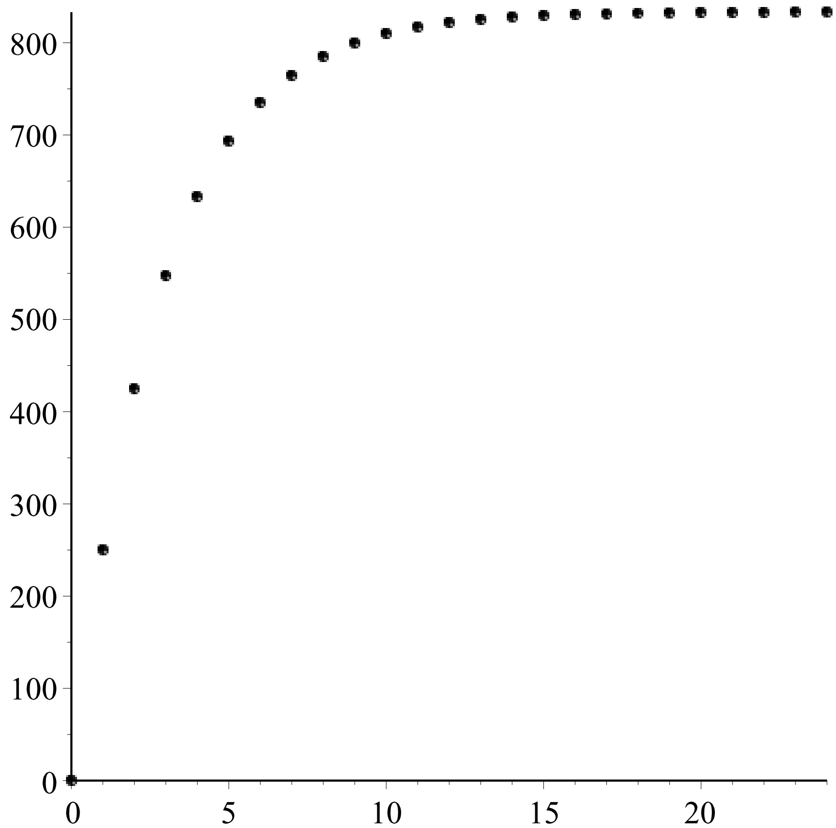

What is the limiting behavior of this dynamical system?

$\lim_{n \to \infty} d(n)$

$$\frac{2500}{3} \tag{6.6}$$

evalf (%)

$$833.3333333 \tag{6.7}$$

Interpretation: It is somewhat evident from the plot that the drug reaches a value where change eventually stops and the concentration in the bloodstream levels at 2500/3 mg. (You may verify this by plugging this value in for *a*(*n*) in the difference equation.) If this dose is safe and effective, then this dosage schedule is acceptable.