Graphically Analyzing Equilibrium Values

CURM Background Material, Fall 2014

In this worksheet, we will graphically analyze the equilibrium values for models of the form $a(n+1) = r \cdot a(n) + b$, where r and b are constant.

For the first portion of our work, let us assume that $b = 50$. So we are looking at $a(n+1) = r \cdot a(n) + 50$. We will look at three cases, $r > 1$, $0 < r < 1$, and $r < 0$. See the class notes for an explanation of why we look at these two cases.

Case 1: *r* > 1

We choose a particular value for r, $r = 2$. So, we have $a(n+1) = 2 \cdot a(n) + 50$. Let us (1) determine the equilibrium value, and (2) look at this graphically in Maple, assuming various initial conditions.

$solve(ae = 2 \cdot ae + 50)$

$$-50 \quad \textbf{(1)}$$

Notice that the equilibrium value is negative. Thus, if we are looking at a model of a physical system, we expect that we will never attain this equilibrium.

$sol1 := rsolve(\{a(n+1) = 2 \cdot a(n) + 50, a(0) = 10\}, a(n))$

$$60\ 2^n - 50 \quad \textbf{(2)}$$

$s1 := unapply(sol1, n)$

$$n \rightarrow 60\ 2^n - 50 \quad \textbf{(3)}$$

$sol2 := rsolve(\{a(n+1) = 2 \cdot a(n) + 50, a(0) = 0\}, a(n))$

$$-50 + 50\ 2^n \quad \textbf{(4)}$$

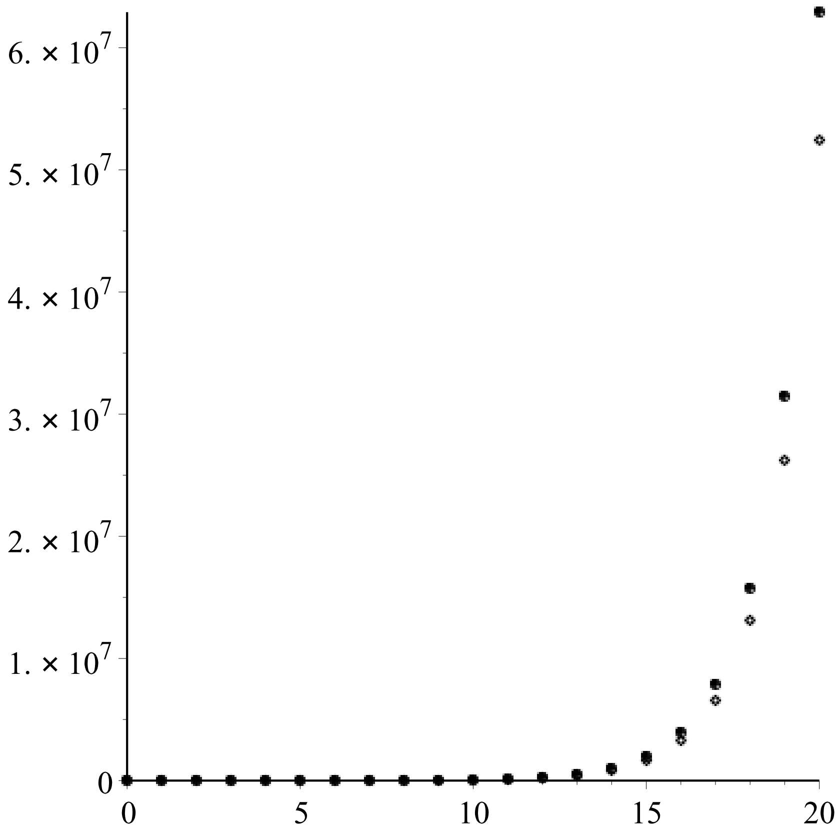
$s2 := unapply(sol2, n)$

$$n \rightarrow -50 + 50\ 2^n \quad \textbf{(5)}$$

$with(plots):$
$p1 := pointplot(\{seq([i, s1(i)], i = 0..20)\}, symbol = solidcircle):$

$p2 := pointplot(\{seq([i, s2(i)], i = 0..20)\}):$

$display(p1, p2)$



Case 2: $0 < r < 1$

In this case, we choose $r = 0.25$. So, we have $a(n+1) = 0.25 \cdot a(n) + 50$. Let us (1) determine the equilibrium value, and (2) look at this graphically in Maple, assuming various initial conditions.

$solve(ae = 0.25 \cdot ae + 50)$

$$66.66666667 \quad \textbf{(6)}$$

Notice that the equilibrium solution is now positive, so there is hope that we may attain this equilibrium value. Look at a solution graphically to see if this appears to happen.

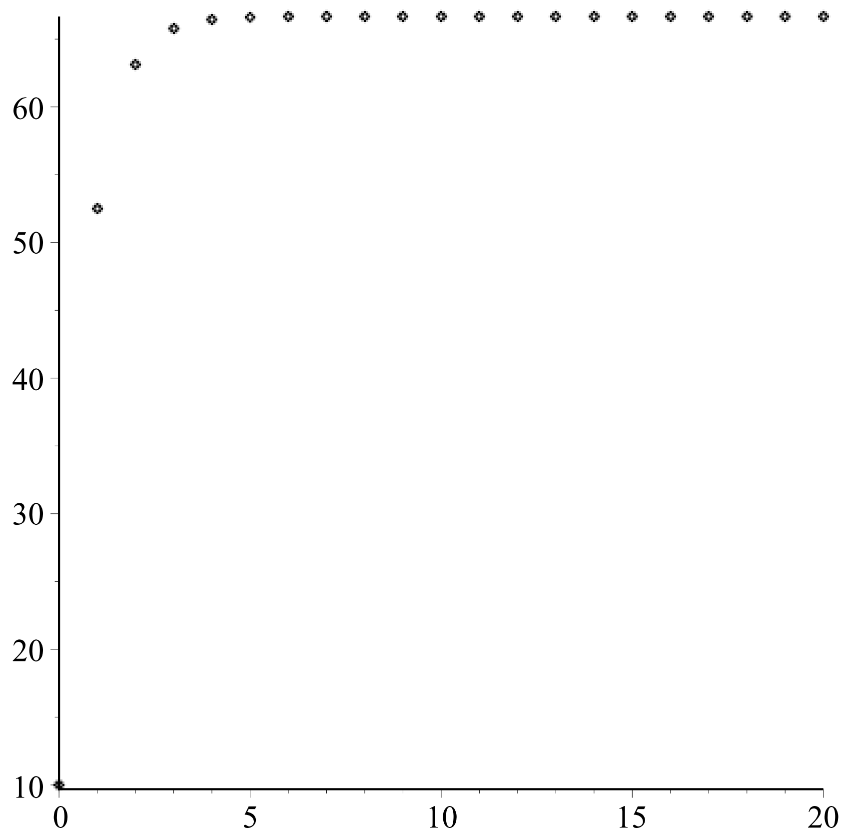
$sol3 := rsolve(\{a(n+1) = 0.25 \cdot a(n) + 50, a(0) = 10\}, a(n))$

$$-\frac{170}{3}\left(\frac{1}{4}\right)^{n}+\frac{200}{3} \quad (7)$$

$s3 := unapply(sol3, n)$

$$n \rightarrow -\frac{170}{3}\left(\frac{1}{4}\right)^{n}+\frac{200}{3} \quad (8)$$

$pointplot(\{seq([i, s3(i)], i=0..20)\})$



Case 3: *r* < 0

In this case, we choose $r = -1.01$. So, we have $a(n+1) = -1.01 \cdot a(n) + 50$. Let us (1) determine the equilibrium value, and (2) look at this graphically in Maple, assuming various initial conditions.

$solve(ae = -1.01 \cdot ae + 50)$

$$24.87562189 \quad (9)$$

Notice that the equilibrium solution is now positive, so there is hope that we may attain this equilibrium value. Look at a solution graphically to see if this appears to happen.

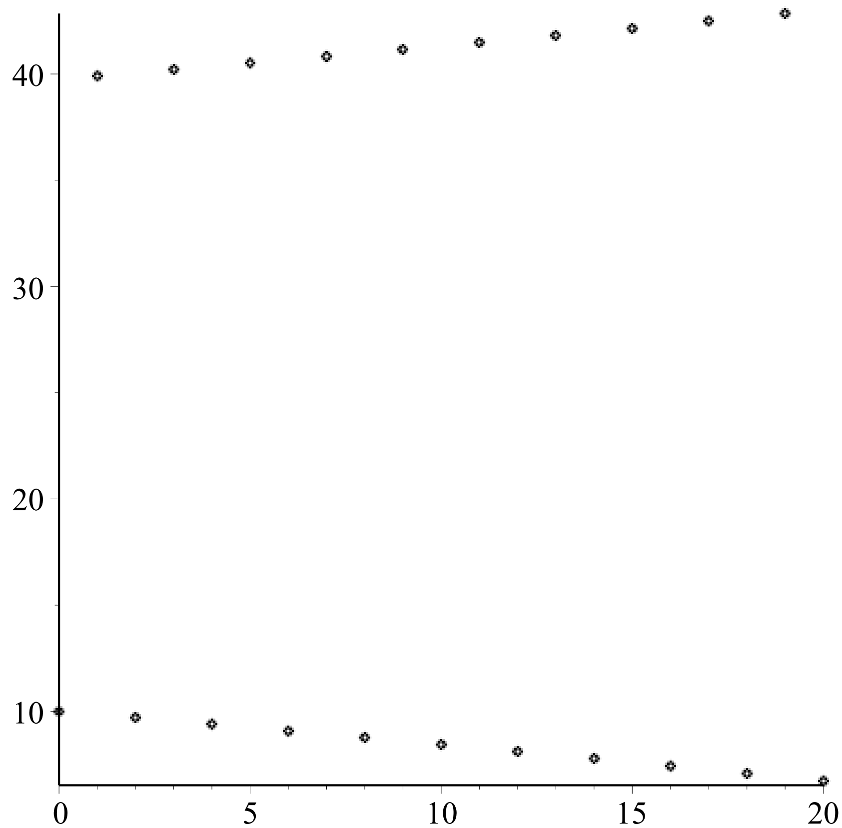
$sol4 := rsolve(\{a(n+1) = -1.01 \cdot a(n) + 50, a(0) = 10\}, a(n))$

$$-\frac{2990}{201}\left(-\frac{101}{100}\right)^n + \frac{5000}{201} \qquad \textbf{(10)}$$

$s4 := unapply(sol4, n)$

$$n \rightarrow -\frac{2990}{201}\left(-\frac{101}{100}\right)^n + \frac{5000}{201} \qquad \textbf{(11)}$$

$pointplot(\{seq([i, s4(i)], i = 0..20)\})$



Notice that the solution oscillates. That this is not unexpected is obvious from the solution, but can also easily be seen by computing the first few iterates.