Example: Analysis of Equilibrium Solutions using Phase Portraits

Commensalism

CURM Background Material, Fall 2014

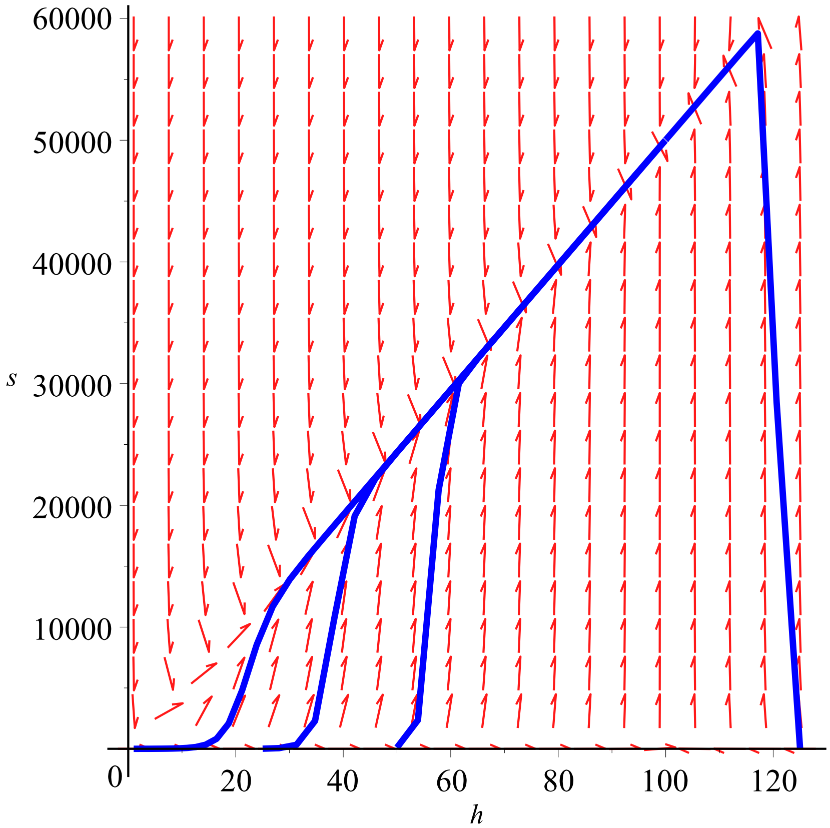
restart

$$solve\left(\left\{0 = 0.5 \cdot h \cdot \left(1 - \frac{h}{100}\right), 0 = 0.01 \cdot s \cdot \left(1 - \frac{s}{25}\right) + 0.002 \cdot s \cdot h, \right\}, \{h, s\}\right)$$

$$\{h = 0., s = 0.\}, \{h = 0., s = 25.\}, \{h = 100., s = 0.\}, \{h = 100., s = 525.\} \tag{1}$$

$with(DEtools)$:

$$DEplot\left(\left[diff(h(t), t) = 0.5 \cdot h(t) \cdot \left(1 - \frac{h(t)}{100}\right), diff(s(t), t) = 0.01 \cdot s(t) \cdot \left(1 - \frac{s(t)}{25}\right) + 0.2 \cdot s(t) \cdot h(t)\right], [h(t), s(t)], t = 0..15, [[h(0) = 1, s(0) = 1], [h(0) = 25, s(0) = 10], [h(0) = 50, s(0) = 100], [h(0) = 125, s(0) = 25]], linecolor = blue\right)$$



It appears from the phase portrait that (100, 50025) is asymptotically stable. Let's look at the linearized system and analyze the stability of this equilibrium solution.

$$f1 := (h, s) \rightarrow 0.5 \cdot h \cdot \left(1 - \frac{h}{100}\right);$$

$$f2 := (h, s) \rightarrow 0.01 \cdot s \cdot \left(1 - \frac{s}{25}\right) + 0.002 \cdot s \cdot h$$

$$(h, s) \rightarrow 0.5\, h \left(1 - \frac{1}{100}\, h\right)$$

$$(h, s) \rightarrow 0.01\, s \left(1 - \frac{1}{25}\, s\right) + 0.002\, s\, h \quad \textbf{(2)}$$

$df1h := unapply(diff(f1(h, s), h), h, s);$
$df1s := unapply(diff(f1(h, s), s), h, s);$

$$(h, s) \rightarrow 0.5 - 0.01000000000\, h$$

$$(h, s) \rightarrow 0 \quad \textbf{(3)}$$

$df2h := unapply(diff(f2(h, s), h), h, s);$
$df2s := unapply(diff(f2(h, s), s), h, s);$

$$(h, s) \rightarrow 0.002\, s$$

$$(h, s) \rightarrow 0.01 - 0.0008000000000\, s + 0.002\, h \quad \textbf{(4)}$$

Evaluate the Jacobian of the functions *f1* and *f2* at the nonzero equilibrium solution and determine its eigenvalues.

$$A := \begin{bmatrix} df1h(100, 525) & df1s(100, 525) \\ df2h(100, 525) & df2s(100, 525) \end{bmatrix}$$

$$\begin{bmatrix} -0.500000000 & 0 \\ 1.050 & -0.2100000000 \end{bmatrix} \tag{5}$$

$with(LinearAlgebra) :$
$Eigenvalues(A)$

$$\begin{bmatrix} -0.2100000000 \\ -0.5000000000 \end{bmatrix} \tag{6}$$