SOLUTIONS

Physics 4C Mid-Term Exam 1, 2024 Spring

(1) What is the critical angle (in degrees) for visible light going from air (with $n_{\text{air}} = 1.00$) into glass (with $n_{\text{glass}} = 1.5$)?

(a) 88.4 (b) 78.6 (c) 41.8 (d) 18.3 (e) The critical angle is not defined.

$n_1 = 1.00$ (air)

$n_2 = 1.5$ (glass)

$\theta_{critical} = \arcsin\left(\frac{1.5}{1.00}\right) =$ undefined

$\Rightarrow$ choice (e)

(2) All convex mirrors have radius of curvature $R < 0$. All convex mirrors therefore have focal length $f < 0$. All convex mirrors have image distance $q < 0$. All mirrors have:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f},$$

where p is the object distance and $p > 0$ always, by convention. All convex mirrors therefore show images that are:

(a) Upright, diminished, and real

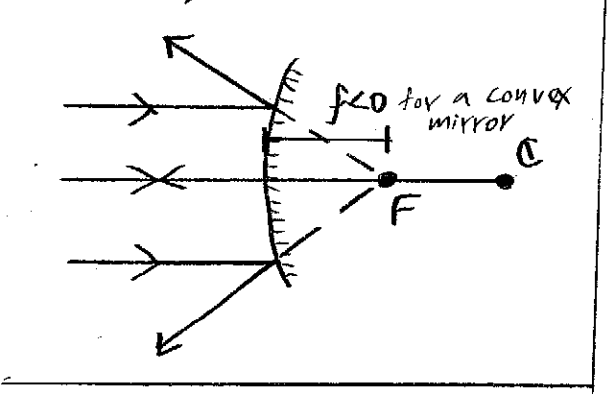
(b) Upright, diminished, and virtual

(c) Inverted, diminished, and real

(d) Inverted, magnified, and virtual

(e) Inverted, diminished, and virtual

By the sign conventions for mirrors, an upright image has $M > 0$. $M = -\frac{q}{p} > 0$, since $q < 0$ and $p > 0$, so the image in a convex mirror is always **upright**.



A diminished image has $|M| < 1$, which means $0 < M < 1$. All convex mirrors have $f < 0$ (by definition), so they have $1/f < 0$, so $\frac{1}{p} + \frac{1}{q} = \frac{1}{f} < 0$. This implies: $\frac{1}{p} < -\frac{1}{q}$, so: $\frac{q}{p} < -1$ and: $-\frac{q}{p} < +1$. Since $M = -\frac{q}{p}$, $M = +1$, and from above, $M > 0$, so $0 < M < 1$. This implies $|M| < 1$, so a convex mirror's image is always **diminished**.

Since $q < 0$, the image must form in back of the mirror, hence **virtual**. $\Rightarrow$ **Choice (b)**

(3) The speed of light changes when it goes from ethyl alcohol, with $n_1 = 1.361$, to carbon tetrachloride, with $n_2 = 1.461$. The ratio of the speeds v_2/v_1 is:
(a) 1.99 (b) 1.07 (c) 0.93 (d) 0.51 (e) 0.76

$n \equiv \frac{c}{v}$, so $\frac{v_2}{v_1} = \frac{c/n_2}{c/n_1} = \frac{n_1}{n_2} = \frac{1.361}{1.461} = 0.93 = n$

$\Rightarrow$ choice (c)

(4) A lens has a front surface with a radius of curvature of -0.2 m, and a back surface that is flat. This lens is made of glass, with $n = 1.5$. What is the focal length (in m) of this lens?
(a) 0.2 (b) -0.3 (c) 0.3 (d) -0.4 (e) 0.4

The lensmaker's equation for a lens with index of refraction n in air is:

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right].$$

Here, $R_1 = -0.2\text{m}$

$R_2 \to \infty$ since flat, $= 0$

so: $\frac{1}{f} = (1.5-1)\left[\frac{1}{-0.2\text{m}} - \frac{1}{\infty}\right]$

$$f = \frac{-0.2\text{m}}{0.5} = f = -0.4\text{m}$$

$\Rightarrow$ choice (d)

(5) A 100 kW radio station emits e/m waves in all directions in a spherical pattern from an antenna on top of a mountain. What is the intensity (in W/m^2) of the signal at a distance of 10 km? Assume the radiation reflected from Earth is negligible. [Hint: The area of a sphere $A = 4\pi r^2$.]

(a) 8×10^{-5} (b) 8×10^{-6} (c) 3×10^{-3} (d) 0.8 (e) 10^{-3}

Find – $I = \frac{\text{Power}}{\text{Area}}$

$P = 100\ \text{kW} = 10^5\ \text{W}$

$r = 10\ \text{km} = 10^4\ \text{m}$

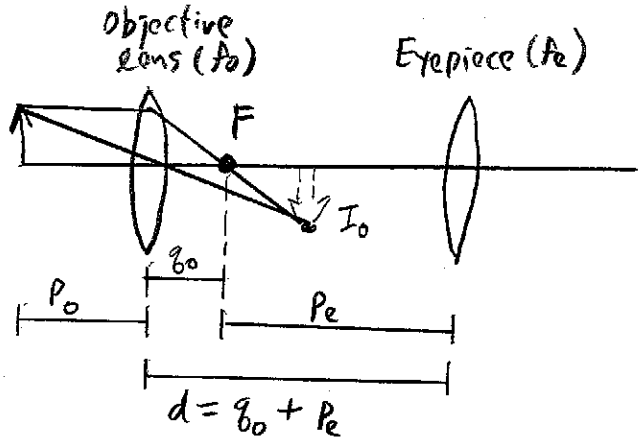
$$I = \frac{P}{A} = \frac{P}{4\pi r^2} = \frac{10^5\ \text{W}}{4\pi (10^4\ \text{m})^2} = \frac{10^5}{4\pi (10^8)}\ \frac{\text{W}}{\text{m}^2}$$

$I = 8 \times 10^{-5}\ \text{W/m}^2$

⇒ Choice (a)

(6) A microscope is made of two lenses. The one in front is called the objective lens, and the one in back is called the eyepiece. The objective lens has $f_0 = +0.90$ cm, and the eyepiece has $f_e = +1.1$ cm. The two lenses are separated by a distance of 10 cm. If an object is 1.0 cm in front of the objective lens, where (in cm) will the final image from the eyepiece be located?

(a) −30 (b) −15 (c) −23 (d) −11 (e) −9



Find – q_e, the image distance from the second lens (the eyepiece).

For the first lens (the objective lens):

$$\frac{1}{p_o} + \frac{1}{q_o} = \frac{1}{f_o},\ \text{so}\ \frac{1}{1.0\text{cm}} + \frac{1}{q} = \frac{1}{+0.90\text{cm}}$$

$\Rightarrow q_o = +9.0$ cm.

$d = 10\text{cm} = q_o + p_e$, so $p_e = 1.0$ cm.

For the second lens (the eyepiece):

$$\frac{1}{p_e} + \frac{1}{q_e} = \frac{1}{f_e},\ \text{so}\ \frac{1}{+1.0\text{cm}} + \frac{1}{q_e} = \frac{1}{+1.1\text{cm}}$$

$\Rightarrow q_e = -11$ cm

⇒ Choice (d)

Box your final answer. No work = No credit on this part.
Problems.

(A) A dedicated sports car enthusiast polishes the inside and outside surfaces of a hubcap that is a section of a sphere.

(a) She holds the hubcap 0.20 m from her face, and looks into the inside surface of the hubcap, which has a radius of curvature of +0.60 m.
(i) (3 points) She sees her image reflected by the inside surface of the hubcap. What is the image distance of this image?
(ii) (3 points) What is the magnification of this image?
(iii) (2 points) Is this a real or a virtual image, and how do you know?
(iv) (2 points) Is this image upright or inverted, and how do you know?

(b) She now flips the hubcap around, so that she looks into the outside surface of the hubcap. Again, she holds the hubcap 0.20 m from her face. The outside surface of the hubcap has a radius of curvature of -0.60 m. Assume the hubcap has negligible thickness, between the front and back surfaces.
(i) (3 points) She sees her image reflected by the outside surface of the hubcap. What is the image distance of this image?
(ii) (3 points) What is the magnification of this image?
(iii) (2 points) Is this a real or a virtual image, and how do you know?
(iv) (2 points) Is this image upright or inverted, and how do you know?

(a) eye ⟷ $p = +0.20\,m$, $R = +0.60\,m$ ⇒ concave mirror

(i) $\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$

$q = \frac{1}{\left(\frac{2}{R} - \frac{1}{p}\right)} = \left[\frac{2}{+0.60\,cm} - \frac{1}{0.20\,m}\right]^{-1}$

$\boxed{q = -0.60\,m}$

(ii) $M = -\frac{q}{p} = \frac{-(-0.60\,m)}{+0.20\,m}$

$\boxed{M = +3}$

(iii) By the sign conventions for mirrors, since $q < 0$, **virtual**.

(iv) Since $M > 0$, **upright**.

(b) eye ⟷ $p = +0.20\,m$, $R = -0.60\,m$ ⇒ convex mirror

(i) $\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$

$q = \frac{1}{\left(\frac{2}{R} - \frac{1}{p}\right)} = \left[\frac{2}{-0.60\,m} - \frac{1}{0.20\,m}\right]^{-1}$

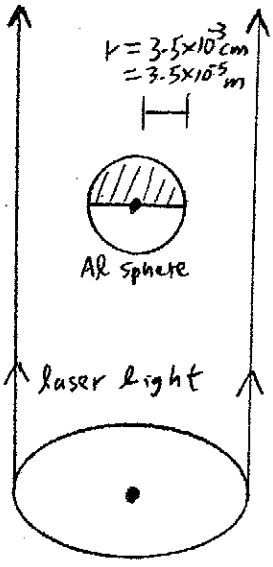
$\boxed{q = -0.12\,m}$

(ii) $M = -\frac{q}{p} = \frac{-(-0.12\,m)}{+0.20\,m}$

$\boxed{M = +0.60}$

(iii) By the sign conventions for mirrors, since $q < 0$, **virtual**.

(iv) Since $M > 0$, **upright**.



(B) (4 points each) A laser with a circular cross section is used to suspend a spherical aluminum bead in Earth's gravitational field, by balancing the force coming from the radiation pressure against the downward gravitational force. The bead has a mass of 1.0×10^{-6} grams and a radius of 3.5×10^{-3} cm. Some helpful information is:

The acceleration of gravity $g = 9.8$ m/s^2. The force of gravity $F_{\text{gravity}} = mg$.

Volume of a sphere $= \frac{4}{3}\pi r^3$. Area of a circle $= \pi r^2$.

(i) What is the radiation intensity needed to support the aluminum bead, assuming the bead reflects all the incident laser light?
(ii) What is the radiation intensity needed to support the aluminum bead, assuming the bead absorbs all the incident laser light?
(iii) If the laser beam has a radius of 0.10 cm, what is the power required for this laser?
(iv) What is the maximum magnetic field of this laser light?
(v) What is the maximum electric field of this laser light?

$F_{gravity} = mg = (1.0\times10^{-6}\,g)\left(\frac{1\,kg}{10^3\,g}\right)\left(9.8\,\frac{m}{s^2}\right) = 9.8\times10^{-9}\,kg\cdot m/s^2$.

This must balance the force exerted by the radiation pressure of the laser upward. Since Pressure = Force/Area,

$$P_{rad} = \frac{F_{gravity}}{A} = \frac{mg}{\pi r^2},$$

since the sphere reflects or absorbs the laser light shining upward over its cross-sectional area, $A = \pi r^2$.

(i) For complete reflection, $P_{rad} = 2I/c$,

so $$I_r = \frac{c\,P_{rad}}{2} = \frac{c(mg)}{2\pi r^2} = \frac{(3.00\times10^8\,\frac{m}{s})(9.8\times10^{-9}\,kg\,m/s^2)}{2\pi(3.5\times10^{-3}\,m)^2}$$

$$\boxed{I_r = 3.8\times10^8\,W/m^2}$$

(ii) For complete absorption, $P_{rad} = I/c$, so $I_a = c\,P_{rad} = \boxed{I_a = 7.6\times10^8\,W/m^2}$,

$= 2\times3.8\times10^8\,W/m^2$.

(iii) I = Power/Area, so Power $= I(\pi R^2)$

For complete reflection, Power $= I_r\,\pi(1.0\times10^{-3}\,m)^2 = 1200\,W$.

For complete absorption, Power $= I_a\,\pi(1.0\times10^{-3}\,m)^2 = 2400\,W$,

so the power required is the greater of these, $\boxed{\text{Power} = 2400\,W.}$

(iv) $I = \frac{E_{max}B_{max}}{2\mu_0}$ and $\frac{E_{max}}{B_{max}} = c$, so $I = \frac{c\,B_{max}^2}{2\mu_0}$ and $B_{max} = \sqrt{\frac{2\mu_0 I}{c}}$,

so: $$B_{max} = \left[\frac{2(4\pi\times10^{-7}\,\frac{Tm}{A})(7.6\times10^8\,\frac{W}{m^2})}{(3.00\times10^8\,m/s)}\right]^{1/2} = \boxed{B_{max} = 2.5\times10^{-3}\,T,}$$

for the maximum B_{max}.

(v) $\frac{E_{max}}{B_{max}} = c$ so $E_{max} = c\,B_{max} = (3.00\times10^8\,\frac{m}{s})(2.5\times10^{-3}\,T)$

$$\boxed{E_{max} = 7.6\times10^5\,V/m}$$