SOLUTIONS to PHYS 4C Mid-Term Exam 2, 2026 Spring

Instructions: Read the problems *carefully* and give the best answer, based on the material presented in class and in the text. The multiple-choice questions are worth 10 points each.

(1) Never shine a green laser at anyone, especially not at an airplane, since the pilots can see the beam. Some fool did this in 2002, and he was arrested on domestic terrorism charges. Suppose that a green laser shines light with $\lambda = 5.32 \times 10^{-7}$ m. This light travels outward from the laser through a circular aperture that is 3.00 mm in diameter. How many meters in diameter is the beam, at a jet airliner altitude of exactly 30,000 feet? (Recall that 1 foot = 0.3048 m.) (a) 3.96 (b) 1.98 (c) 1.62 (d) 0.0030 (e) 0.0016

Diffraction through a circular aperture –

$\theta_{min} \simeq 1.22 \frac{\lambda}{D}$

Find – y

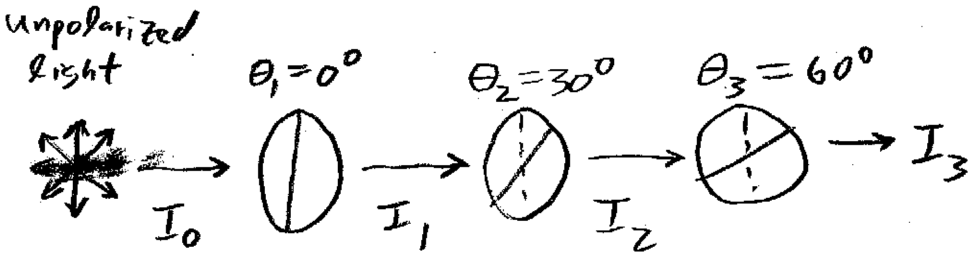
$$\frac{y}{h} \simeq \theta_{min} \simeq 1.22 \frac{\lambda}{D}$$

$$y \simeq \frac{1.22 \lambda h}{D} = \frac{(1.22)(5.32 \times 10^{-7}\,m)(30{,}000\,\cancel{ft})\left(\frac{0.3048\,m}{\cancel{ft}}\right)}{3.00 \times 10^{-3}\,m}$$

$$\boxed{y \simeq 1.98\,m}$$

$\boxed{\rightarrow \text{choice (b)}}$

(2) Unpolarized light is passed through three successive Polaroid filters, each with its transmission axis at 30.0° to the preceding filter. What percentage of the light gets through?
(a) 0 (b) 9.375 (c) 28.125 (d) 31.64 (e) 56.25



Find – $(I_3/I_0) \times 100\%$

$I_1 = \frac{1}{2} I_0$ since I_0 is unpolarized.

$$I_2 = I_1 \cos^2(\theta_2 - \theta_1) = \frac{1}{2} I_0 \cos^2(30° - 0°) = \frac{1}{2} I_0 \cos^2(30°)$$

$$I_3 = I_2 \cos^2(\theta_3 - \theta_2) = \frac{1}{2} I_0 \cos^2(30°) \cos^2(60° - 30°)$$

$$= \frac{1}{2} I_0 \cos^2(30°) \cos^2(30°) = \frac{1}{2} I_0 \cos^4(30°)$$

$$(I_3/I_0) \times 100\% = \left[\frac{1}{2} \cos^4(30°)\right] \times 100\%$$

$$\boxed{(I_3/I_0) \times 100\% = 28.125\,\%} \boxed{\rightarrow \text{choice (c)}}$$

(3) Your body temperature is 98.6° F = 37.0° C = 310 K. Assuming that your skin is a perfect thermal radiator (with $\epsilon = 1$), determine the wavelength of the maximum intensity of electromagnetic radiation that you radiate (in microns, where 1 micron = 10^{-6} m). (a) 8.0 (b) 9.3 (c) 3.0 (d) 5.7 (e) 29.4

Wien's law for thermal radiation —

$$\lambda_{max}\,(m) = \frac{2.898 \times 10^{-3}\, m \cdot K}{T(K)}$$

Find — λ_{max}

$$\lambda_{max} = \left(\frac{2.898 \times 10^{-3}\, \cancel{m} \cdot \cancel{K}}{310\, \cancel{K}}\right)\left(\frac{1 \text{ micron}}{10^{-6}\, \cancel{m}}\right)$$

$$\boxed{\begin{array}{l} \lambda_{max} = 9.3 \text{ microns} \\ \hline \rightarrow \text{choice (b)} \end{array}}$$

(4) Microscopes are inherently limited in resolution by the wavelength they use. How much smaller detail can be resolved, with an electron microscope that uses electrons that have been accelerated through a potential difference of 50,000 V, than with a visible-light microscope that uses red light, with $\lambda = 5.0 \times 10^{-7}$ m? (Recall that $U = qV$, where U = electric potential energy, q = electron charge and V = the potential difference, assume non-relativistic motion, and as always, pick the closest choice to the answer.)
(a) 1000 (b) 10,000 (c) 100,000 (d) 1 million (e) 100 trillion

For non-relativistic motion,

$$p = mv \quad \text{and} \quad K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$\Delta K = \Delta U = qV = \frac{p^2}{2m}$$

$$\Rightarrow p = \sqrt{2mqV}$$

For an electron microscope,

$$\lambda_e = \frac{h}{p} = \frac{h}{\sqrt{2m_e q_e V}}$$

since the de Broglie wavelength $\lambda = h/p$.

$$= \frac{(6.626 \times 10^{-34}\, J \cdot s)}{\left[(2)(9.11 \times 10^{-31}\, kg)(1.602 \times 10^{-19}\, C)(50{,}000\, V)\right]^{1/2}}$$

$$\lambda_e = 5.48 \times 10^{-12}\, m$$

For the visible-light microscope, $\lambda_r = 5.0 \times 10^{-7}$ m.

Since $\theta_{min} \simeq \frac{\lambda}{D} \left(\simeq 1.22 \frac{\lambda}{D}\right)$ for both microscopes,

$$\frac{\lambda_r}{\lambda_e} \sim \frac{\theta_r}{\theta_e} \sim \frac{5.0 \times 10^{-7}\, m}{5.48 \times 10^{-12}\, m}$$

$$\boxed{\begin{array}{l} \dfrac{\lambda_r}{\lambda_e} \sim 10^5 \\ \hline \rightarrow \text{choice (c)} \end{array}}$$

(5) The longest wavelength of electromagnetic radiation that the silicon detector in an everyday, commercial-grade digital camera can detect is 1 micron = 10^{-6} m. Find the work function of silicon, in eV.
(a) 0.34 (b) 0.58 (c) 1.17 (d) 1.24 (e) 5.92

The photoelectric effect, for which Einstein won his Nobel prize –

$$K_{max} = \frac{hc}{\lambda} - \Phi \qquad \text{Find} - \Phi$$

$$\Phi = \frac{hc}{\lambda} - K_{max}$$

At cutoff, $K_{max} = 0$,

$$\text{So } \Phi = \frac{hc}{\lambda} = \frac{(1240 \text{ eV-nm})}{(10^{-6} m)(1 nm / 10^{-9} m)}$$

$$\Phi = \frac{1240 \text{ eV}\cdot\cancel{nm}}{10^3 \cancel{nm}} = \boxed{\Phi = 1.24 \text{ eV}} \boxed{\rightarrow \text{choice (d)}}$$

(6) A relativistic electron (with $v > 0.1$ c) has a kinetic energy equal to twice its rest energy. Determine its speed. (a) 0.76 c (b) 0.81 c (c) 0.94 c (d) 0.54 c (e) 0.87 c

Find – v of the electron, v_e.

Relativistic ($v > 0.1 c$)
kinetic energy $K_{rel} = (\gamma - 1) m c^2 = 2 m_e c^2$, here

$$(\gamma - 1)\cancel{m_e}\cancel{c^2} = 2\cancel{m_e}\cancel{c^2}$$

$$(\gamma - 1) = 2$$

$$\gamma = 3 = \frac{1}{\sqrt{1-(v_e/c)^2}}$$

$$1 - (v_e/c)^2 = 1/3^2 = 1/9$$

$$(v_e/c)^2 = 8/9$$

$$v_e/c = \sqrt{8/9}$$

$$\boxed{v_e = 0.94 c} \boxed{\rightarrow \text{choice (c)}}$$

Problems: Box your final answer. No work = No credit

(A) (i) (4 points) A satellite is in a low-Earth orbit at an altitude $L = 200$ km $= 2.0 \times 10^5$ m above Earth's surface. Its camera has a circular aperture (which is a diameter, not a radius) of $D = 2.5$ meters. This camera can detect visible light, which has a wavelength of 550 nm (where 1 nm $= 10^{-9}$ m). If the camera has adaptive optics, it can compensate for the blurring effects of Earth's atmosphere. However, diffraction imposes an absolute limit on the camera's image resolution, or in other words, the smallest detail the camera can see. How many meters in diameter (y) is the smallest object on Earth that this camera can resolve, or see clearly? [Hint: Since the satellite is many kilometers from Earth, $\sin\theta \approx y/L$.]

(ii) (4 points) This satellite also has an infrared camera. It can detect thermal radiation (also called blackbody radiation, or Planck radiation), from objects that are not transparent, such as rocks, cars, or buildings. Suppose one of these objects radiates the maximum intensity of its thermal radiation at a wavelength $\lambda_{max} = 10$ microns, where 1 micron $= 1.0 \times 10^{-6}$ m. What is the temperature of this object, in Kelvins?

(iii) (4 points) Suppose a thermal radiator of any Kelvin temperature T (not necessarily the radiator mentioned in the previous problem) radiates with a total intensity I. If this thermal radiator then doubles in Kelvin temperature, what will its total intensity I' now be, as a function of I? Assume the area of this thermal radiator is the same, at both Kelvin temperatures.

(iv) (4 points) A transparent coating on the windshield of a car has an index of refraction $n = 1.25$. The purpose of the coating is to reflect light away from the windshield, to reduce unwanted stray light seen by the driver, which is called glare. The glass under this coating has $n = 1.50$. The coating is 101 nm thick. What is the wavelength (in nm) of the light that is most strongly reflected?

(v) (4 points) Suppose an electron is moving at a speed of 0.999 999 90 c = 0.99999990 c. (That's seven instances of the numeral "9.") At what speed, expressed as a fraction of the speed of light in empty space c, does a proton have the same linear momentum as this electron? The proton mass $m_p = 1836\, m_e$, where m_e is the mass of the electron. The masses of the electron and the proton are listed in the Physics 4C Formula List, at the end of this exam.

(A)

(i) For a satellite looking down at Earth,

$$\theta_m \simeq \frac{Y}{L}.$$

By the Rayleigh criterion,

$$\theta_{min} \simeq \frac{1.22\,\lambda}{D},$$

for diffraction through a circular aperture.

Find - Y

So: $\frac{Y}{L} \simeq \frac{1.22\lambda}{D}$ $\lambda = 550$ nm $= 5.50 \times 10^{2} \times 10^{-9}$ m $= 5.50 \times 10^{-7}$ m

and so:

$$Y \simeq \frac{1.22\lambda L}{D} = \frac{(1.22)(5.50 \times 10^{-7}\,\text{m})(2.0 \times 10^{5}\,\text{m})}{(2.5\,\text{m})}$$

$$\boxed{Y \simeq 5.4 \times 10^{-2}\,\text{m} = 5.4\,\text{cm}}$$

(ii) Find - T, for thermal radiation.

Wien's law is $\lambda_{max}(\text{m}) = \frac{2.898 \times 10^{-3}\,\text{m}\cdot\text{K}}{T(\text{K})}$,

So $T(\text{K}) = (2.898 \times 10^{-3}\,\text{m}\cdot\text{K})/(1.0 \times 10^{-5}\,\text{m})$

$$\boxed{T = 290\,\text{K}}$$

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(A)

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(iii) For thermal radiators at any T, $I = \sigma T^4$, where σ (sigma) $= 5.67\times10^{-8}\ \frac{W}{m^2 K^4}$, the Stefan-Boltzmann constant.

$$\frac{I'}{I} = \frac{\cancel{\sigma} T'^4}{\cancel{\sigma} T^4}, \text{ so } I' = (T'/T)^4 I, \text{ and with } T' = 2T,$$

$$\boxed{I' = 16I}, \text{ since } 2^4 = 16.$$

(iv) Air $n_1 = 1.00$ / Coating $n_2 = 1.25$ / Glass $n_3 = 1.50$

This is thin-film interference. $t = 101\text{ nm} = 1.01\times10^{-7}\text{ m}$.

To maximize the reflection, we want destructive interference, so:

$$t = (m + \tfrac{1}{2})\frac{\lambda}{2n} = \frac{\lambda}{4n} \text{ for } m = 0.$$

$$\lambda = 4nt = (4)(1.25)(101\text{ nm}) = \boxed{\lambda = 505\text{ nm} = 5.05\times10^{-7}\text{ m}}.$$

(v) At relativistic speeds (with $v > 0.1c$), $p = \gamma m v$, NOT $p = mv$.

For the proton, $p_p = \gamma_p m_p v_p = \gamma_e m_e v_e = p_e$, for the electron, since even in special relativity, linear momentum is conserved.

$$\gamma_p v_p = \left(\frac{m_e}{m_p}\right)\gamma_e v_e = \underbrace{\left(\frac{0.511\text{ MeV}/c^2}{938.272\text{ MeV}/c^2}\right)}_{=1/1836}\frac{(0.999\,999\,90\,c)}{\sqrt{1-(0.999\,999\,90)^2}}$$

$\gamma_p v_p = 1.2178c \equiv Ac$, letting $A = 1.2178$. Find v_p.

$$\frac{v_p}{\sqrt{1-(v_p/c)^2}} = Ac$$

$$\frac{v_p^2}{1-(v_p/c)^2} = A^2c^2$$

$$v_p^2 = A^2c^2 - A^2c^2(v_p/c)^2$$

$$(v_p/c)^2 = A^2 - A^2(v_p/c)^2$$

$$(v_p/c)^2[1 + A^2] = A^2$$

$$\left(\frac{v_p}{c}\right)^2 = \frac{A^2}{1+A^2} = \frac{1.483}{1+1.483} = \frac{1.483}{2.483}$$

$$\frac{v_p}{c} = \sqrt{\frac{1.483}{2.483}} = \boxed{\frac{v_p}{c} = 0.7783}$$

(B) A particle of mass m is trapped inside a one-dimensional box of width L. One wall of the box is at $x = 0$ and the other is at $x = L$. The particle is the second excited state (with $n = 3$) and has the wavefunction $\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{3\pi x}{L}\right)$.

(i) (4 points) At what three locations is there a maximum probability for observing the particle? At what two locations *inside* the box is there a minimum probability of observing the particle? Give the values of x for all cases. [Hint: you may solve this graphically.]

(ii) (4 points) What is the probability that this particle will be between $0 \le x \le \frac{L}{4}$? Use the integral $\int sin^2(ax)\, dx = \frac{x}{2} - \frac{sin(2ax)}{4a}$. Give the value and the explicit analytical solution, or in other words, the worked integration for this problem: you get zero points if you write just, "The probability is obviously..." or, "My calculator shows that the probability is..."

(iii) (4 points) If the particle makes a transition from $n = 3$ to the ground state, where $n = 1$, what is the energy of the photon that is emitted? Write your answer in terms of m and L.

(iv) (4 points) A photon with a wavelength of 5.0×10^{-11} m is scattered straight backward from an initially stationary electron, because of the Compton effect. What is the wavelength of the scattered photon, in meters?

(v) (4 points) Two radio antennas are separated by a distance $d = 1$ km. Both antennas are $h = 50$ m tall. See the figure ~~below.~~ on the following page.

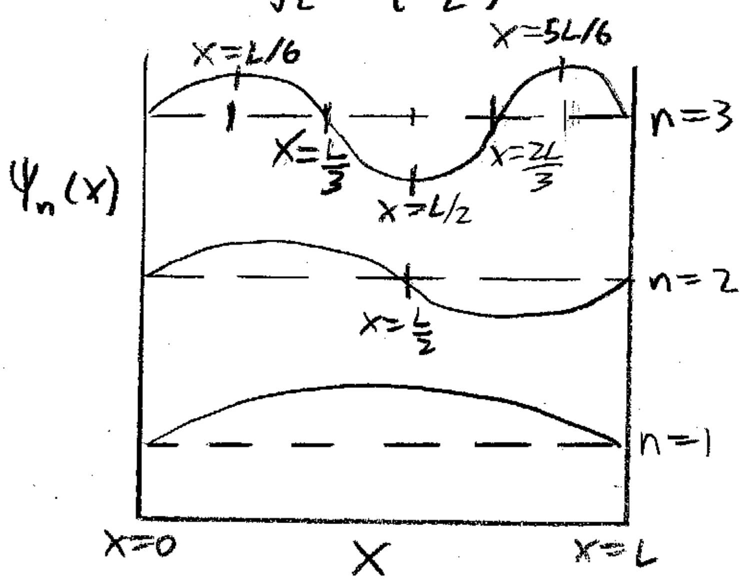
All the radio energy is radiated from the top of the antennas. Both antennas simultaneously transmit identical signals at the same wavelength. A car is traveling on the ground on a path that is a straight line between the two antennas. A radio in the car receives the signals. Neglect the effect of the curvature of Earth, since it is relatively small here (with the horizon curving downward by 7.85 cm per kilometer, or 8 inches per mile).

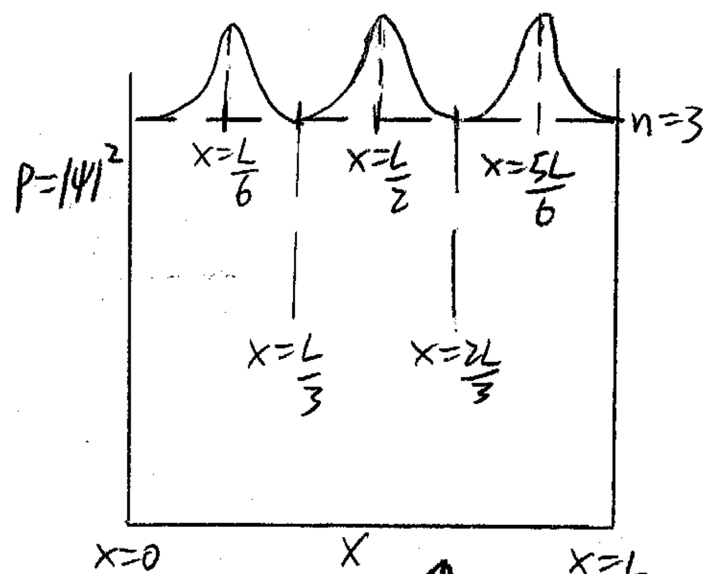
When the car is 1/3 of the way between the antennas, it is at the position of the maximum with $m = 2$. What is the frequency of the signals?

(B)

(i) This can be solved graphically, by plotting $\psi_n(x)$ and $p = |\psi(x)|^2$.

$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{3\pi x}{L}\right)$





For the probability density $p = |\psi(x)|^2$, when $n = 3$, (shown above)

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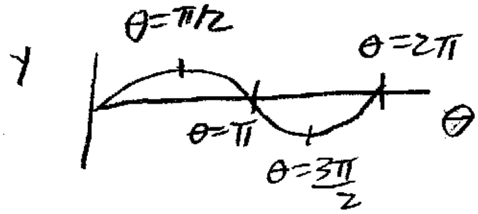
the maxima are at $\boxed{x = L/6,\ x = L/2, \text{ and } x = 5L/6.}$

The minima are at $\boxed{x = L/3 \text{ and } x = 2L/3}$.



, so $\sin\left(\frac{3\pi}{2}\right) = -1.$

(B)

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(ii) $P_{ab} = \int_{x=a}^{x=b} |\psi(x)|^2 dx$, so between $0 \le x \le L/4$,

$$P_{ab} = P = \int_{x=0}^{x=L/4} \left(\frac{2}{L}\right) \sin^2\left(\frac{3\pi x}{L}\right) dx = \frac{2}{L}\int_{x=0}^{L/4} \sin^2(ax)\,dx, \text{ where } a = \frac{3\pi}{L}.$$

$$P = \left(\frac{2}{L}\right)\left[\frac{x}{2} - \frac{\sin(2ax)}{4a}\right]_{x=0}^{x=L/4} = \left(\frac{2}{L}\right)\left[\frac{x}{2} - \frac{L\sin\left(\frac{6\pi x}{L}\right)}{12\pi}\right]_{x=0}^{x=L/4}$$

$$= \left(\frac{2}{L}\right)\left[\left(\frac{L/4}{2}\right) - \frac{L\sin\left(\frac{6\pi L}{4L}\right)}{12\pi} - \cancel{\left(\frac{0}{2}\right)}^{=0} + \cancel{\frac{L\sin(0)}{12\pi}}^{=0}\right]$$

$$= \left[\frac{1}{4} - \frac{\sin(3\pi/2)}{6\pi}\right] = \left[\frac{1}{4} + \frac{1}{6\pi}\right] = \boxed{P = 0.30305}$$

since $\sin(3\pi/2) = -1$.

(iii) For a particle confined to a box with $0 \le x \le L$,

$$E_n = \left(\frac{h^2}{8mL^2}\right) n^2, \quad n = 1, 2, 3 \dots$$

when the particle jumps from $n=3$ (the second excited state) to $n=1$ (the ground, or lowest-energy state), to conserve energy it emits a photon with energy E equal to the energy difference between the two states: $E = \left(\frac{h^2}{8mL^2}\right)[3^2 - 1^2] = \frac{8h}{8mL^2} = \boxed{E = \frac{h^2}{mL^2}}$

(iv) This is the Compton effect with $\theta = 180^0$ (which is straight backward),

so: $\lambda' - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta) = \frac{(6.626\times10^{-34}\,J\cdot s)(1-(-1))}{(9.11\times10^{-31}\,kg)(3.00\times10^{8}\,m/s)}$

$$\lambda' = 5.0\times10^{-11}\,m + 4.9\times10^{-12}\,m = \boxed{\lambda' = 5.5\times10^{-11}\,m}$$

(v) See next page

(B)
(V) Constructive interference – $\delta = r_2 - r_1 = m\lambda$, with $m = 2$ here.
Find – $f = c/\lambda$.

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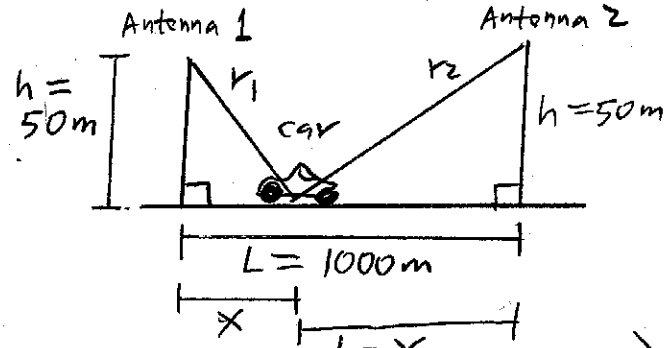


$$\delta = r_2 - r_1 = 2\lambda$$
$$= \sqrt{h^2 + (L-x)^2} - \sqrt{h^2 + x^2} = 2\lambda$$
$$2\lambda = \sqrt{(50m)^2 + \left[\frac{2(1000m)}{3}\right]^2} - \sqrt{(50m)^2 + \left(\frac{1000m}{3}\right)^2}$$
$$\lambda = \frac{331.47m}{2} = 165.7\ m$$
$$f = c/\lambda = (3.00\times10^8\ m/s)/(165.7m)$$
$$\boxed{f = 1.81\times10^6\ Hz = 1.81\ MHz,}$$
with $1 Hz = 1\frac{cycle}{s} = 1 s^{-1}$.

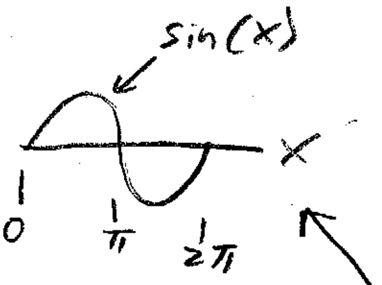
An alternative, analytical (not graphical) solution to (B)(i) is:

$$P = |\psi(x)|^2 = \frac{2}{L}\sin^2\left(\frac{3\pi x}{L}\right)$$

To find the maxima and minima of a function of x, set $\frac{dP}{dx} = 0$ and solve for x –

$$\frac{dP}{dx} = \frac{4}{L}\left[\sin\left(\frac{3\pi x}{L}\right)\cos\left(\frac{3\pi x}{L}\right)\right]\left(\frac{3\pi}{L}\right)$$
$$= \left(\frac{12\pi}{L^2}\right)\left[\sin\left(\frac{3\pi x}{L}\right)\cos\left(\frac{3\pi x}{L}\right)\right],$$

so $\sin\left(\frac{3\pi x}{L}\right)\cos\left(\frac{3\pi x}{L}\right) = 0.$



If $\sin\left(\frac{3\pi x}{L}\right) = 0$, $\arcsin(0) = \frac{3\pi x}{L} \Rightarrow \arcsin(0) = 0, \pi$, and 2π, over one cycle.

Exclude $x = 0$ and $x = L$, since $\psi = 0$ at $x = 0$ and $x = L$ anyway, since these are the edges of the box.

$\Rightarrow \pi = \frac{3\pi x}{L} \Rightarrow$ $\boxed{x = \frac{L}{3} \text{ and } x = \frac{2L}{3} \text{ are the minima}}$.

To find the maxima, $\cos\left(\frac{3\pi x}{L}\right) = 0$ $\Rightarrow x = L/6$, $x = L/2$, and $x = 5L/6$.