2-D DOA Estimation of Coherent Wideband Signals with Auxiliary-Vector Basis

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***Abstract*—We develop a two-dimensional (2-D) direction-of-arrival (DOA) estimation scheme for coherent wideband source signals using coherent signal subspace method based auxiliary-vector (CSSM-AV) basis. Computation of the basis is carried out by a modified version of the orthogonal CSSM-AV filtering algorithm. The proposed method reconstructs the signal subspace using a cross-correlation matrix after which the modified CSSM-AV algorithm is employed to estimate the azimuth and elevation angles. Then, successive orthogonal maximum cross-correlation auxiliary vectors are calculated to form a basis for the scanner-extended signal subspace. This technique is very efficient in reducing the algorithm complexity. Since it does not require that the eigenvectors be determined in order to find the signal subspace and yields a superior resolution performance for closely spaced sources even when the number of samples is low. Specifically, the complexity of the proposed 2-D DOA estimation algorithm compared to the CSSM algorithm is more favorable when the number of signals arriving on the antenna element is much less than the number of antenna elements. Performance evaluation shows that the proposed method outperforms competing methods such as CSSM, TOPS and WAVES algorithms in terms of estimation error, probability of resolution and number of sample support for a given SNR in scenarios in which many sources are present in the system, the array size is large, and the number of samples is small.**

***Index Terms*—Wideband direction-of-arrival (DOA) estimation, uniform circular arrays (UCA), auxiliary-vector (AV) filtering, small sample support.**

I. Introduction

Direction-of-arrival (DOA) estimation with sensor arrays has been a central topic of signal processing research over the past few decades due to its importance in radar, sonar, and wireless communications [1], [2], [3]. DOA estimation techniques can be classified into two main categories; maximum-likelihood (ML) methods, which are based on the maximization of the probability density function of the received signal and signal subspace methods that are based on the eigen-decomposition of the autocovariance matrix of the received signal. Two of the well known subspace algorithms are multiple signal characterization (MUSIC) [4] and estimation of signal parameters via rotational invariance technique (ESPRIT) [5]. In general, ML-type algorithms have superior performance compared to subspace-based techniques when either the signal-to-noise (SNR) ratio or the sample size is small. Also, in the case of correlated signal sources the performance of subspace-based estimators degrades significantly, as compared to ML schemes. The trade-off of ML-type algorithms compared to subspace-based is the high computational complexity.

These DOA algorithms are primarily designed for narrowband signal sources and thus cannot be used for wideband signals, as the phase difference between sensor outputs is dependent on both the DOA and on the temporal frequency. A number of wideband DOA estimation algorithms are proposed, which are mainly based on coherent or noncoherent wideband techniques. The incoherent signal subspace method (ISSM) [6], [7] is one of the simplest wideband DOA estimation method. In [7], the authors propose the ISSM for wideband signals in which the received wideband signal is decomposed into a set of narrowband signals on different frequency subbands using the discrete Fourier transform (DFT), so that high-resolution narrowband DOA estimators such as MUSIC can be applied in each subband. In ISSM, each frequency subband is processed independently and then the separate results are averaged over all subbands. ISSM methods work well in favorable situations, i.e., high SNR and well-separated signals. Its performance may deteriorate with coherent sources or SNR variations in different frequency subbands. An outlier in any frequency subband could severely degrade the final estimate due to the averaging process.

To overcome this problem a number of improved methods have been proposed. Among those are the coherent signal subspace method (CSSM) [8]. In CSSM, the received wideband signals are first decomposed into a set of narrowband signals similar to ISSM. The covariance matrices of each subband are transformed into covariance matrices of a certain focusing frequency by multiplying them with focusing matrices. The focusing matrices are used for the alignment of the signal subspaces of narrowband components within the bandwidth of the signals, followed by the averaging of narrowband covariance matrices into a universal covariance matrix. Then, any narrowband DOA estimators such as MUSIC can be applied to the universal covariance matrix to obtain the DOA estimates. There are a number of different methods designed for focusing matrices in the conventional CSSM. Although some of those methods are simple to implement, they require initial DOA estimates to calculate the focusing matrices. Therefore, the final DOA estimates are very sensitive to the initial estimates. The weighted average of signal subspace (WAVES) technique [9] is a widely used method, which also requires the use of

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focusing matrices. Even though WAVES can avoid the initial-value requirement by beamforming invariance techniques, its performance is worse than the conventional CSSM that has a good initial estimate and the existence of the beamforming matrix depends on the size of a field of view (FOV) and the array geometry.

The TOPS technique [10] stands for test of orthogonality of projected subspaces, is a relatively new method which is essentially ISSM and estimates DOA through tests of orthogonality between projected signal subspaces and noise subspaces whose performance is more favorable compared to the conventional ISSM. TOPS algorithm first applies singular value decomposition (SVD) to get the signal subspace f_0 of one frequency point that is present in the bandwidth of every source. Based on this signal subspace, a matrix is constructed that can transform the subspace of one frequency and one DOA to another frequency and another DOA. Then, the orthogonality of projected signal subspaces and noise subspaces of every DOA and every frequency are tested. In this processing it is obvious that f_0 plays a crucial role, as the estimate error of f_0 can strongly affect the overall performance of the algorithm. Moreover, it has the problem of detecting false peaks in the pseudo spectrum and the performance deteriorates in the low SNR regime or when having coherent sources.

In this paper, we propose a CSSM based auxiliary-vector (AV) [11] subspace method coined as CSSM-AV for two-dimensional DOA estimation for coherent wideband source signals. The advantage of AV based algorithm is that it does not require eigen-decomposition, hence it is favorable in terms of algorithm complexity and its DOA estimation performance has been reported to be superior in terms of resolution, DOA estimation error for a given SNR [11]. Specifically, the complexity of AV compared to the existing DOA methods such as CSSM algorithm is more favorable when the number of signals arriving on the antenna element is much less than the number of antenna elements. One additional benefit of the proposed method is that it can even operate under low sample support, short data record size.

Performance evaluation demonstrates that the proposed method has very good resolving capability, especially, when the angle separation is very small, e.g. two degrees at low SNR regime. In addition to that the proposed method provides favorable probability of resolution compared to TOPS, WAVES and CSSM algorithms at low SNR regime.

The remainder of this paper is organized as follows. We first present the system model in Section II. We then discuss the conventional CSSM in Section III, after which we present the proposed subspace auxiliary-vector algorithm in Section IV. In Section V, we present and compare the computational complexity of the proposed algorithm with other competing algorithms. In Section VI, we discuss the performance evaluation of the proposed algorithm. Finally, in Section VII, we draw the main conclusions.

II. System Model

In this paper, we consider uniform circular array (UCA), consisting of M isotropoic and identical antenna elements distributed uniformly over a circle with radius $R = \lambda/2$, where λ is the wavelength. The phase azimuth angle of m^{th} element is $\phi_m = \frac{2\pi m}{M}$ with $m = 1, 2, ..., M$. K signals arriving on the UCA with directions $(\theta_1, \phi_1), (\theta_2, \phi_2), ..., (\theta_K, \phi_K)$. The received signal for the m^{th} sensor can be expressed as

$$x_m(t) = \sum_{k=1}^{K} s_k\left(t - \tau_m(\theta_k, \phi_k)\right) + n_m(t),\ m = 0, 1, ..., M-1, \tag{1}$$

where $s_k(t)$ is the k^{th} source signal, $n_m(t)$ is the noise observed at m^{th} sensor and $\tau_m(\theta_k, \phi_k)$ is the relative delay. The sampled signal at the m^{th} sensor is decomposed into N narrowband components, $X_m(f_n)$, $n = 0, 1, 2, ..., N-1$. We can express (1) in the frequency domain as

$$X_m(f_n) = \sum_{k=1}^{K} S_k(f_n) e^{-j2\pi f_n \tau_m(\theta_k, \phi_k)} + N_m(f_n), \tag{2}$$

where $S_k(f_n)$ is the n^{th} frequency component of the source signal $s_k(t)$ and $N_m(f_n)$ is the n^{th} frequency component of the noise $n_m(t)$. In matrix vector form (2) is expressed as

$$\mathbf{x}(f_n) = \mathbf{A}(\theta, \phi, f_n)\mathbf{s}(f_n) + \mathbf{n}(f_n), \tag{3}$$

where $\mathbf{A}(\theta, \phi, f_n) = [\mathbf{a}(\theta_1, \phi_1, f_n), \mathbf{a}(\theta_2, \phi_2, f_n), ..., \mathbf{a}(\theta_K, \phi_K, f_n)]$ is the full-rank $M \times K$ matrix of location vectors. The steering vectors $\mathbf{a}(\theta_k, \phi_k, f_n)$ of m^{th} sensor at frequency f_n can be expressed as

$$a_m(\theta_k, \phi_k, f_n) = e^{j2\pi f_n \frac{R}{\lambda} \sin(\theta_k) \cos\left(\frac{2\pi m}{M} - \phi_k\right)}. \tag{4}$$

It is assumed that the signal and noise samples are independent identically-distributed (i.i.d.) sequences of complex Gaussian random vectors with unknown source and noise covariance matrices, $\mathbf{R}_s(f_n)$ and $\sigma^2\mathbf{I}$, respectively. With these assumptions, the covariance matrix of the observation vector at the frequency f_n is given by

$$\mathbf{R}_x(f_n) = \mathbf{A}(\theta, \phi, f_n)\mathbf{R}_s(f_n)\mathbf{A}^H(\theta, \phi, f_n) + \sigma^2\mathbf{I}, \tag{5}$$

where superscript H denotes Hermitian transpose - that is, transpose conjugate.

III. Coherent Signal Subspace Method (CSSM)

Before we present our CSSM-AV based approach it is essential to introduce the theoretical background of the preprocessing step of the CSSM algorithm [8]. The main idea is to demonstrate on how to combine the signal-subspace at different frequencies with the objective to generate a single signal subspace that is representative of all the sources and angle of arrivals. For notational simplicity, we drop the frequency, angles of elevation and azimuth variables and represent $\mathbf{A}(\theta, \phi, f_n)$ by $\mathbf{A}_n$, $\mathbf{x}(f_n)$ by $\mathbf{x}_n$ and so forth. The new observation vector $\mathbf{y}_n$, as discussed in [8], can be written as

$$\mathbf{y}_n = \mathbf{T}_n\mathbf{x}_n, \tag{6}$$

where the $\mathbf{T}_n$'s are called the *focusing matrices*. The idea is to align or focus the signal space at all frequency bins into a common bin at a reference frequency, f_0 such that

$$\mathbf{T}_n \mathbf{A}_n = \mathbf{A}_0,\ n = 0, ..., N-1, \tag{7}$$

note that $\mathbf{T}_0$ is the identity matrix, $\mathbf{I}$.

The focusing matrices, $\mathbf{T}_n$, $n = 0, ..., N-1$, are selected from

$$\begin{aligned} &\min_{\mathbf{T}_n} ||\mathbf{A}_0 - \mathbf{T}_n \mathbf{A}_n||, \\ &\text{subject to } \ \mathbf{T}_n^H \mathbf{T}_n = \mathbf{I}, \end{aligned} \tag{8}$$

where $||\cdot||$ is the Frobenius matrix norm. The solution to this minimization is given by [12],

$$\mathbf{T}_n = \mathbf{V}_n \mathbf{W}_n^H, \tag{9}$$

where $\mathbf{V}_n$ and $\mathbf{W}_n$ are the left and the right singular matrices of $\mathbf{A}_0 \mathbf{A}_n^H$. One of the most challenging issues in the coherent signal-subspace method is the choice of the best focusing frequency, f_0, to decrease the estimation bias. We use the method described by Valaee and Kabal in [13], which has been shown to be the optimal method for the selection of focusing subspace selection in the CSSM algorithm. Initial DOA estimates are required to compute the focusing matrices. Obviously, a bad estimate will affect the system's performance. In practice, the unknown covariance matrix is estimated by the sample covariance matrix using transformed observation vectors, $\mathbf{y}_n$ at the n^{th} frequency bin

$$\hat{\mathbf{R}}_y(f_n) = \frac{1}{L} \sum_{i=1}^{L} \left\{ \mathbf{y}_n \mathbf{y}_n^H \right\}, \tag{10}$$

where L is the number of snapshots. An average of these aligned sample covariance matrices gives a universal sample covariance matrix, which is expressed as

$$\mathbf{R}_y = \frac{1}{N} \sum_{n=1}^{N} \hat{\mathbf{R}}_y(f_n) \tag{11}$$

that can be used for DOA estimation. After estimating the universal covariance matrix $\mathbf{R}_y$'s we are ready to apply them to the proposed CSSM-AV algorithm.

IV. Subspace Auxiliary-Vector Algorithm

Having the covariance matrix $\mathbf{R}_y$, we present noneigenvector basis that spans the signal subspace extended in dimension by the DOA scanner vector $\mathbf{s}_{\theta,\phi}$, $\theta \in (0^\circ, 90^\circ)$ and $\phi \in (0^\circ, 360^\circ)$. For the notation simplicity, we can fix the azimuth angle, ϕ and scan in the direction of elevation, θ by scanning vector, $\mathbf{s}_\theta$. According to the work in [11], the initial basis vector, $\mathbf{v}_0(\theta)$ is defined as

$$\mathbf{v}_0(\theta) \triangleq \frac{\mathbf{R}_y \mathbf{s}_\theta}{\|\mathbf{R}_y \mathbf{s}_\theta\|}. \tag{12}$$

Having defined $\mathbf{v}_0(\theta)$, the auxiliary vector, $\mathbf{g}_1(\theta)$ is determined by maximizing the magnitude of the statistical cross-correlation between the $\mathbf{v}_0^H(\theta)\mathbf{y}_n$ and $\mathbf{g}_1^H(\theta)\mathbf{y}_n$, subject to the orthonormality constraints $\mathbf{g}_1^H(\theta)\mathbf{v}_0(\theta) = 0$ and $\mathbf{g}_1^H(\theta)\mathbf{g}_1 = 1$, as follows:

$$\begin{aligned} \mathbf{g}_1(\theta) &= \underset{\substack{\mathbf{g}_1(\theta) \\ \mathbf{g}_1^H(\theta)\mathbf{v}_0(\theta)=0, \|\mathbf{g}_1(\theta)\|=1}}{\arg\max} \left| E\left\{ \mathbf{v}_0^H(\theta) \mathbf{y}_n \mathbf{y}_n^H \mathbf{g}_1(\theta) \right\} \right| \\ &= \underset{\substack{\mathbf{g}_1(\theta) \\ \mathbf{g}_1^H(\theta)\mathbf{v}_0(\theta)=0, \|\mathbf{g}_1(\theta)\|=1}}{\arg\max} \left| \mathbf{v}_0^H(\theta) \mathbf{R}_y \mathbf{g}_1(\theta) \right|. \end{aligned} \tag{13}$$

The solution to this constrained optimization problem is obtained as

$$\mathbf{g}_1(\theta) = \frac{\left(\mathbf{I} - \mathbf{v}_0(\theta)\mathbf{v}_0^H(\theta)\right) \mathbf{R}_y \mathbf{v}_0(\theta)}{\left\| \left(\mathbf{I} - \mathbf{v}_0(\theta)\mathbf{v}_0^H(\theta)\right) \mathbf{R}_y \mathbf{v}_0(\theta) \right\|}. \tag{14}$$

The recursion for $k = 2, 3, ..., K-1$ orthonormal auxiliary vectors can be shown as in [11] to be

$$\mathbf{g}_k(\theta) = \frac{\left(\mathbf{I} - \sum_{i=k-2}^{k-1} \mathbf{g}_i(\theta)\mathbf{g}_i^H(\theta)\right) \mathbf{R}_y \mathbf{g}_{k-1}(\theta)}{\left\| \left(\mathbf{I} - \sum_{i=k-2}^{k-1} \mathbf{g}_i(\theta)\mathbf{g}_i^H(\theta)\right) \mathbf{R}_y \mathbf{g}_{k-1}(\theta) \right\|}, \tag{15}$$

where $\mathbf{g}_0(\theta) = \mathbf{v}_0(\theta)$. Then the K^{th} unnormalized auxiliary vector $\mathbf{g}_K(\theta)$ is expressed as

$$\mathbf{g}_K(\theta) = -\mu_{K-1}(\theta) \left(\mathbf{I} - \sum_{i=K-2}^{K-1} \mathbf{g}_i(\theta)\mathbf{g}_i^H(\theta) \right) \mathbf{R}_y \mathbf{g}_{K-1}(\theta), \tag{16}$$

where

$$\mu_k(\theta) = -\mu_{k-1}(\theta) \frac{\mathbf{g}_k^H(\theta)\mathbf{R}_y\mathbf{g}_{k-1}(\theta)}{\mathbf{g}_k^H(\theta)\mathbf{R}_y\mathbf{g}_k(\theta)},\ k = 2, 3, ..., K-1. \tag{17}$$

After the computation of the K orthogonal auxiliary-vector basis $\{\mathbf{v}_0(\theta), \mathbf{g}_1(\theta), ..., \mathbf{g}_K(\theta)\}$ the DOA estimation procedure is done as follows. Define $\theta^{(n)} = n\triangle^\circ$, where $n = 1, 2, ..., \frac{90^\circ}{\triangle^\circ}$ and $\triangle^\circ$ is the angle search step size in degrees and, without loss of generality, assume $90^\circ/\triangle^\circ$ is an integer. We can form a subspace matrix, $\mathbf{S}(\theta^{(n)})$ as,

$$\mathbf{S}(\theta^{(n)}) = \left[\mathbf{v}_0(\theta^{(n)}), \mathbf{g}_1(\theta^{(n)}), ..., \mathbf{g}_K(\theta^{(n)}) \right]. \tag{18}$$

Finally, using (16) and (18) the DOA estimation is computed

$$P_{AV}(\theta^{(n)}) = \frac{1}{\left\| \mathbf{g}_K^H(\theta^{(n)}) \mathbf{S}(\theta^{(n-1)}) \right\|},\ n = 2, 3, ..., \frac{90^\circ}{\triangle^\circ}. \tag{19}$$

V. Complexity Analysis

In Table I, we present the computational complexity analysis for CSSM, TOPS, WAVES and the proposed CSSM-AV algorithm, discussed in Section IV. As we can see, CSSM, and TOPS algorithms have a high computational complexity $O(M^3)$, where M is the number of antenna elements. Since both CSSM and TOPS algorithms algorithms need matrix inversion and eigen-decomposition. The computational complexity of the WAVES algorithm, on the other hand, depends on L narrowband snapshots, N frequency subbands (i.e., NL number of wideband samples) and on M. Therefore, for the number of wideband samples being less than the number of antenna elements (i.e., $NL < M$) the computational complexity

Table I
COMPUTATIONAL COMPLEXITY COMPARISON

Algorithms	Complexity	Main procedures
CSSM	$O(M^3)$	Eigen-decomposition (grid search)
TOPS	$O(M^3)$	Eigen-decomposition
WAVES	$O(NM^2L)$	Eigen-decomposition
Proposed CSSM-AV	$O\left(360°(90°/\triangle°)\,KM^2\right)$	Construction of signal subspace (grid search)

of the WAVES algorithm is relatively smaller compared to the CSSM and TOPS algorithms. The computational complexity of the proposed CSSM-AV algorithm is $O(KM^2)$ per test angle or $O\left(360°(90°/\triangle°)\,KM^2\right)$. In comparison to all of the above algorithms the proposed CSSM-AV algorithm is less costly; especially, when the number of signals arriving on the antenna element is much less than the number of antenna elements. The inherit benefit of the proposed algorithm lies in the fact that it does not require eigen-decomposition, which has a $O(M^3)$ computational complexity.

VI. PERFORMANCE EVALUATION

In this section, we present computer simulation results using MATLAB to demonstrate the effectiveness of the proposed method in the 2-D DOA estimation of coherent wideband signals. We evaluate the performance of four methods: the proposed CSSM-AV, TOPS, WAVES and CSSM. The following simulation constraint condition are made for the tests; i.e., seven-sensor uniform circular array with radius of $R = \lambda/100$ are made. This distance is smaller than traditional $\lambda/2$ distance. Three far-field uncorrelated, and equipowered with wideband sources are placed at $\theta = 10°$, $14°$, $45°$, and $\phi = 23°$, $23°$, $23°$, where θ and ϕ are the elevation and azimuth angles, respectively. The wideband source signal are generated using some of sinusoids with random magnitude and random phase,

$$s(t) = a(t) \sum_{n=1}^{N} e^{j(2\pi f_n t + \phi_n)}, \qquad (20)$$

where the amplitude $a(t)$ is Rayleigh random variable and the phase ϕ_n is uniformly distributed in $[-\pi, \pi]$. The sampling frequency is three times the highest frequency. The sensor output was divided into $L = 100$ snapshots and each snapshots is $N = 256$ samples, which are converted to frequency domain by a 256-point DFT. The statistical performance was evaluated by performing Monte Carlo runs for each scenario.

First, we would like to demonstrate DOA performance for the proposed CSSM-AV and the conventional CSSM method, latter is based on the MUSIC algorithm. In Figs. 1 and 2, we plot the signal output power of the CSSM algorithm and the proposed wideband DOA estimation algorithm, respectively. The number of angle of arrivals, K = 2, $\theta = 10°$, $36°$, and $\phi = 20°$, $44°$. As we can see from Figs. 1 and 2 the conventional CSSM and the proposed CSSM-AV algorithms can clearly detect the angle of elevation and azimuth.

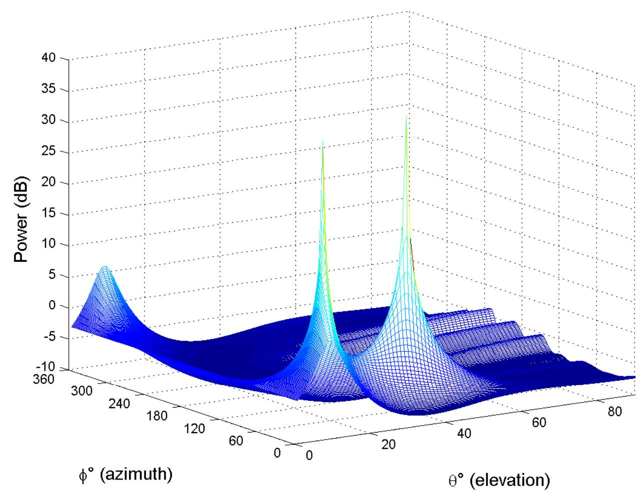

Figure 1. Output power of the CSSM algorithm versus azimuth and elevation.

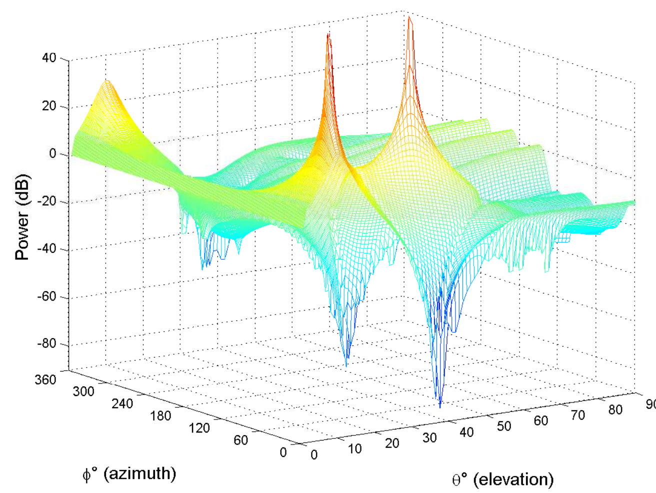

Figure 2. Output power of the proposed CSSM-AV wideband DOA estimation algorithm versus azimuth and elevation.

For the sake of presentation purposes we can show the DOA performance by fixing azimuth angle (ϕ) and focus on the elevation angles (θ). Therefore, in Fig. 3, we plot the power versus elevation. The three angle of arrivals are set at $10°$, $14°$ and $45°$ with azimuth fixed at $23°$, $23°$ and $23°$, respectively. The SNR is set to $13dB$. In this specific example, radius $R = 0.9\lambda$. We can see that all of the four algorithms are able to detect the elevation angles. However, as we can observe the TOPS algorithm may require higher power compared to the rest.

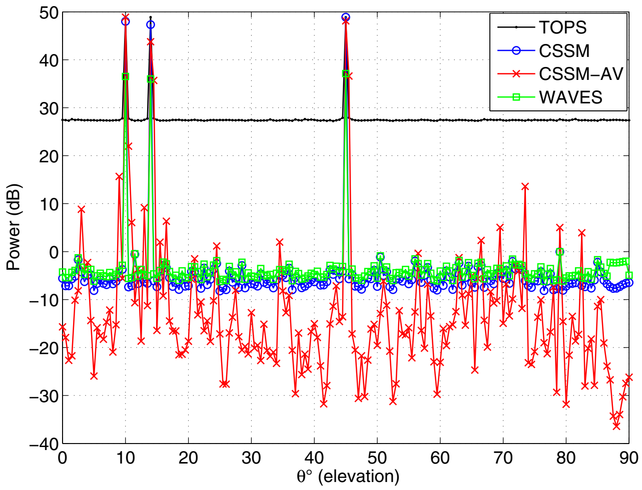

Figure 3. Output power versus elevation (azimuth is fixed, angle separation $4°$).

In Fig. 4, we plot the signal output power of the algorithm as a function of angle of arrival. The SNR is set to $5dB$. The first two angle of arrivals are set at $10°$ and $14°$, i.e., with very close angle of arrival separation, $4°$. We can observe that when the two angle of arrivals are very closed together, i.e., $4°$, the TOPS algorithm fails to accurately detect the angle of arrivals. The proposed CSSM-AV algorithm has slightly better performance compared to both conventional CSSM and WAVES algorithms, which have similar performance.

The probability of resolution is the probability that the algorithm can identify the two closely separated transmitters located at close elevations. In Fig. 5, we plot the probability of resolution versus SNR for the different algorithms. The transmitters are located at elevations of $10°$ and $12°$, respectively. The azimuth is fixed at $23°$, while the elevations are set at $10°$, $12°$, and $45°$. We can observe that TOPS is inferior in performance compared to the rest. While the WAVES and CSSM algorithms can work well at slightly higher SNR regime compared to the proposed algorithm. The proposed CSSM-AV algorithm has stronger resolution capabilities even for low SNR regime, i.e., it can perform well even when the SNR is as low as $5dB$.

In Fig. 6, we plot the root-mean-square error (RMSE) versus SNR (elevation is fixed, angle separation $2°$). We can see that TOPS is the least competitive in performance followed by the WAVES algorithm. CSSM has similar RMSE performance compared to the proposed CSSM-AV and it slightly outperforms CSSM-AV at a few instances in lower SNR regime,

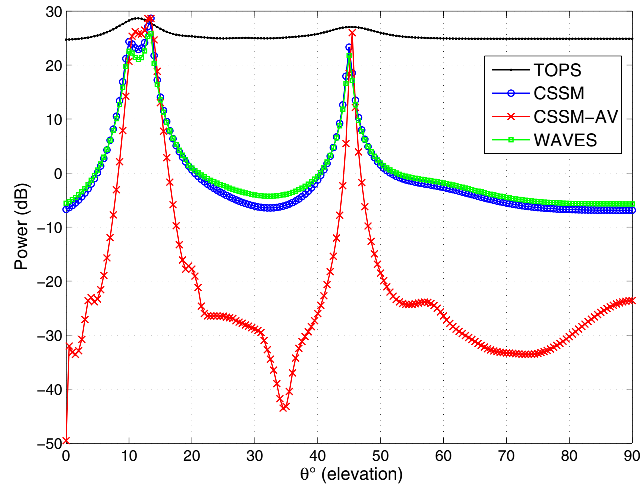

Figure 4. Output power versus elevation (azimuth is fixed, angle separation $4°$).

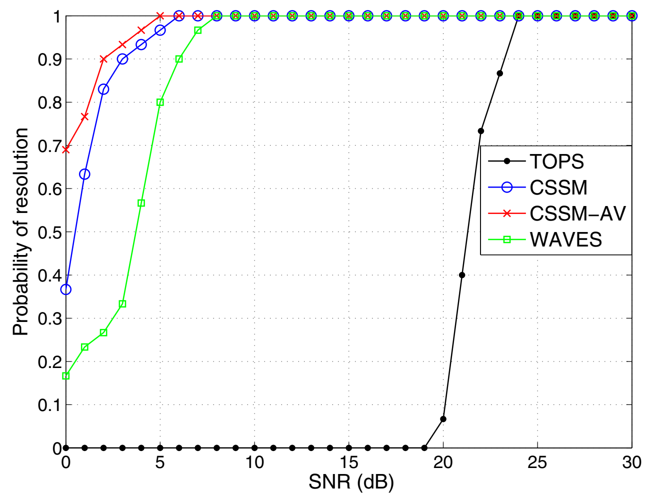

Figure 5. Probability of resolution versus SNR (azimuth is fixed, angle separation $2°$).

while the performance of WAVES is slightly worse than the CSSM.

In Fig. 7, we plot the probability of resolution versus number of samples (for the two DOA separation $10°$ and $12°$). The azimuth is fixed at $23°$, while the elevations are set at $10°$, $12°$, and $45°$. We can observe that TOPS is worst in performance while the WAVES and CSSM algorithms can work well in slightly higher number of samples compared to the proposed algorithm. On the other hand, as we can see the CSSM-AV algorithm is slightly better than CSSM in resolution capabilities even for low number of samples.

In Fig. 8, we plot the RMSE versus number of samples (elevation is fixed, angle separation $2°$). We can see that TOPS is the least competitive in performance followed by WAVES then CSSM algorithms. Both CSSM and WAVES algorithms perform slightly worse than the proposed CSSM-AV algorithm in terms of RMSE for different number of samples.

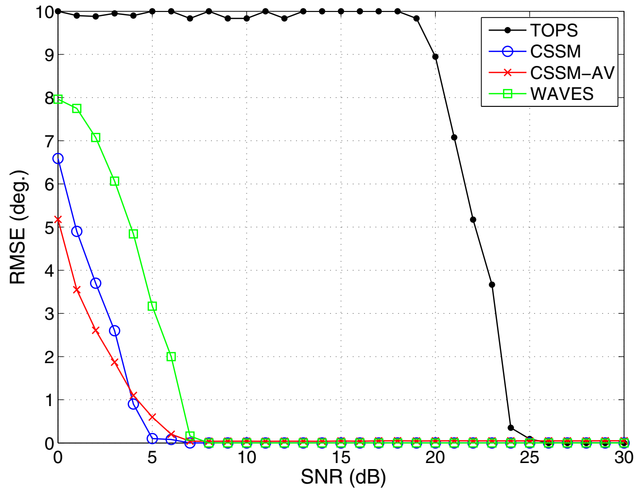

Figure 6. RMSE versus SNR (elevation is fixed, angle separation 2°).

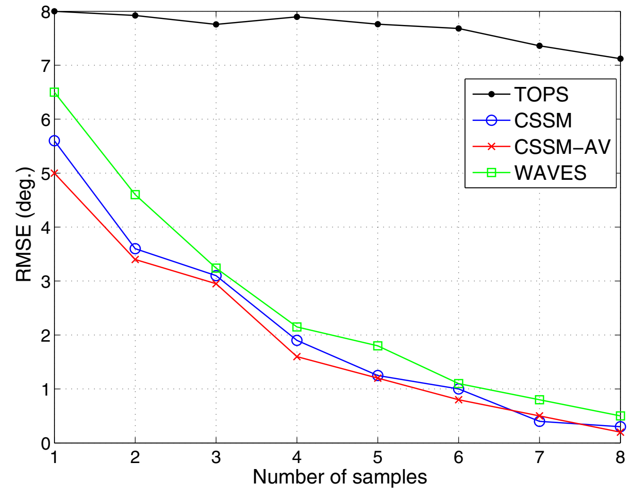

Figure 8. RMSE versus number of samples (elevation is fixed, angle separation 2°).

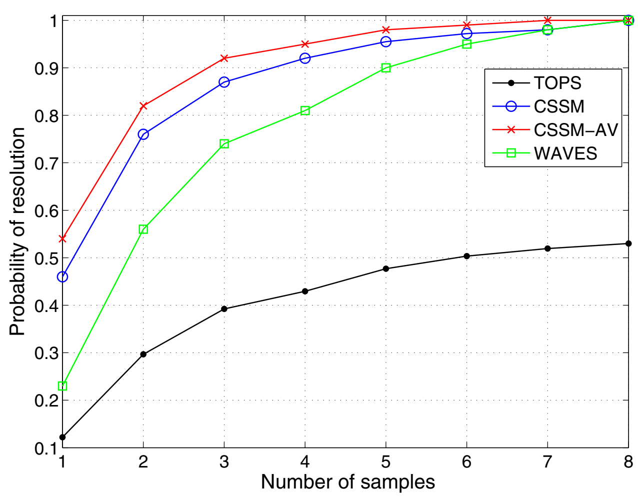

Figure 7. Probability of resolution versus number of samples (azimuth is fixed, angle separation 2°).

VII. Conclusion

In this paper, we proposed a 2-D DOA estimation scheme for coherent wideband source signals using coherent signal subspace method based auxiliary vector basis. The proposed CSSM-AV method is superior in terms of computational complexity compared to other competing subspace methods such as CSSM, TOPS and WAVES algorithms that are heavily dependent on decomposition techniques, while the proposed method does not require eigen-decomposition. We also showed through simulation results that the proposed method has favorable resolving capabilities compared to the other competing algorithms (e.g. TOPS, CSSM and WAVES algorithms) even when the two arrival signal angle separations are small (e.g., 2°) and even when the number of samples is low. The proposed algorithm is shown to have similar resolving capabilities compared to the CSSM algorithm, although the proposed algorithm outperforms in terms of computational cost for scenarios in which many sources are present in the system, the array size is large and the number of samples is small.

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