

1. The domain of the function $f(x) = \frac{3x}{\sqrt{x+2}}$ is

- (a) \mathbb{R}
- (b) $x \geq -2$
- (c) $x > -2$
- (d) $x \neq 2$

$x + 2$ must be greater than or equal to 0 because it is under a square root. But $x + 2$ cannot be 0 because that would force division by 0. Therefore $x + 2 > 0$, or $x > -2$.

2. $\lim_{t \rightarrow 2} \frac{t}{|t-2|} =$

- (a) 2
- (b) ∞
- (c) $-\infty$
- (d) does not exist.

As t approaches 2 from both the right and the left, $\frac{t}{|t-2|}$ becomes more and more positive (the numerator and denominator are both positive, and the denominator gets closer and closer to 0). Therefore the limit is ∞ .

3. The graph of the function $f(x) = \frac{x+3}{x+3}$

- (a) is identical to the graph of $f(x) = 1$
- (b) has a vertical asymptote at $x = -3$
- (c) has a hole at $x = -3$
- (d) has a vertical tangent at $x = -3$

$\frac{x+3}{x+3} = 1$ whenever $x \neq -3$. So the graphs are identical except at $x = -3$, where $\frac{x+3}{x+3}$ is undefined.

4. If the velocity of a bicycle at time $t = 30$ is $\lim_{h \rightarrow 0} \frac{(30 + h)^2 - \sqrt{30 + h} - (30^2 - \sqrt{30})}{h}$ feet per second, then the distance traveled by the bicycle after t seconds could be
- (a) $t^2 - \sqrt{t}$
- (b) $2t - \frac{1}{2}t^{-1/2}$
- (c) $t^2 - \sqrt{30}$
- (d) $t^2 + \sqrt{t}$

Velocity is the derivative of distance. Therefore, if the velocity of the bicycle at $t = 30$ is equal to the above limit, then the distance traveled after t seconds is a function whose derivative is equal to that limit, or $s(t) = t^2 - \sqrt{t}$.

5. For the graph of $f(x)$ shown at right,*
- (a) $f(x)$ is continuous and differentiable at $x = 2$
- (b) $f(x)$ is continuous but not differentiable at $x = 2$
- (c) $f(x)$ is differentiable but not continuous at $x = 2$
- (d) $f(x)$ is neither differentiable nor continuous at $x = 2$

* Please see me for the picture of the graph.

The graph of $f(x)$ shows a cusp (corner) at $x = 2$, so it is continuous but not differentiable there.

6. If $f(-3) = 1$, $f(2) = 1$, $g(2) = -3$, and $g(1) = -1$, then $(g \circ f)(2) =$
- (a) 1
- (b) 2
- (c) -3
- (d) -1

$$(g \circ f)(2) = g(f(2)) = g(1) = -1.$$

7. The height in feet at time t (in seconds) of a ball thrown upward is $s(t) = 50t - 16t^2$. The average velocity of the ball during the first 2 seconds is
- (a) 36 ft./s
- (b) 18 ft./s
- (c) -14 ft./s
- (d) -7 ft./s

Average velocity is distance traveled divided by time elapsed, or

$$\begin{aligned} \frac{s(2) - s(0)}{2 - 0} &= \frac{[50 \cdot 2 - 16 \cdot 2^2] - [50 \cdot 0 - 16 \cdot 0^2]}{2} \\ &= \frac{100 - 64}{2} = 18. \end{aligned}$$

8. $\lim_{x \rightarrow 1} \frac{x-1}{x^2+x-2} =$

(a) $\boxed{\frac{1}{3}}$

(b) $-\frac{1}{3}$

(c) 0

(d) does not exist.

$\frac{x-1}{x^2+x-2} = \frac{x-1}{(x-1)(x+2)}$, which is equal to $\frac{1}{x+2}$ whenever $x \neq 1$. Since the value of the limit does not depend on what happens *at* $x = 1$, only *near* $x = 1$, the value of the limit, using the limit laws, is $\lim_{x \rightarrow 1} \frac{1}{x+2} = \frac{1}{3}$.

9. For the graph of $f(x)$ shown at right*, which of the following could be a graph of $f'(x)$?

* Please see me for an explanation of this question.

BONUS. (5 points) If $f(x) = \begin{cases} 5 & x > 3 \\ |2x - 1| & x \leq 3 \end{cases}$ then the domain of $f'(x)$ is

(a) \mathbb{R}

(b) $x \neq \frac{1}{2}$

(c) $x \neq 3$

(d) $\boxed{x \neq \frac{1}{2}, x \neq 3}$

(e) none of these.

The graph of $f(x)$ consists of three straight lines as shown below. The derivative $f'(x)$ is not defined at $x = \frac{1}{2}$ and $x = 3$. Therefore the domain of $f'(x)$ is all real numbers except $\frac{1}{2}$ and 3.

