

1. If $f(-3) = 1$, $f(2) = 1$, $g(2) = -3$, and $g(1) = -1$, then $(g \circ f)(2) =$

- (a) 1
- (b) 2
- (c) -3
- (d) -1

$$(g \circ f)(2) = g(f(2)) = g(1) = -1.$$

2. The domain of the function $f(x) = \frac{3x}{\sqrt{x+2}}$ is

- (a) \mathbb{R}
- (b) $x \geq -2$
- (c) $x > -2$
- (d) $x \neq 2$

$x + 2$ must be greater than or equal to 0 because it is under a square root. But $x + 2$ cannot be 0 because that would force division by 0. Therefore $x + 2 > 0$, or $x > -2$.

3. $\lim_{t \rightarrow 2} \frac{t}{|t-2|} =$

- (a) 2
- (b) ∞
- (c) $-\infty$
- (d) does not exist.

As t approaches 2 from both the right and the left, $\frac{t}{|t-2|}$ becomes more and more positive (the numerator and denominator are both positive, and the denominator gets closer and closer to 0). Therefore the limit is ∞ .

4. The graph of the function $f(x) = \frac{x+3}{x+3}$
- (a) is identical to the graph of $f(x) = 1$
 - (b) has a vertical asymptote at $x = -3$
 - (c) has a hole at $x = -3$
 - (d) has a vertical tangent at $x = -3$

$\frac{x+3}{x+3} = 1$ whenever $x \neq -3$. So the graphs are identical except at $x = -3$, where $\frac{x+3}{x+3}$ is undefined.

5. If the velocity of a bicycle at time $t = 30$ is $\lim_{h \rightarrow 0} \frac{(30+h)^2 - \sqrt{30+h} - (30^2 - \sqrt{30})}{h}$ feet per second, then the distance traveled by the bicycle after t seconds could be
- (a) $t^2 - \sqrt{t}$
 - (b) $2t - \frac{1}{2}t^{-1/2}$
 - (c) $t^2 - \sqrt{30}$
 - (d) $t^2 + \sqrt{t}$

Velocity is the derivative of distance. Therefore, if the velocity of the bicycle at $t = 30$ is equal to the above limit, then the distance traveled after t seconds is a function whose derivative is equal to that limit, or $s(t) = t^2 - \sqrt{t}$.

6. For the graph of $f(x)$ shown at right,*
- (a) $f(x)$ is continuous and differentiable at $x = 2$
 - (b) $f(x)$ is continuous but not differentiable at $x = 2$
 - (c) $f(x)$ is differentiable but not continuous at $x = 2$
 - (d) $f(x)$ is neither differentiable nor continuous at $x = 2$

* Please see me for the picture of the graph.

The graph of $f(x)$ shows a cusp (corner) at $x = 2$, so it is continuous but not differentiable there.

7. $\lim_{x \rightarrow 1} \frac{x-1}{x^2+x-2} =$
- (a) $\frac{1}{3}$
 - (b) $-\frac{1}{3}$
 - (c) 0
 - (d) does not exist.

$\frac{x-1}{x^2+x-2} = \frac{x-1}{(x-1)(x+2)}$, which is equal to $\frac{1}{x+2}$ whenever $x \neq 1$. Since the value of the limit does not depend on what happens at $x = 1$, only near $x = 1$, the value of the limit, using the limit laws, is $\lim_{x \rightarrow 1} \frac{1}{x+2} = \frac{1}{3}$.

8. The height in feet at time t (in seconds) of a ball thrown upward is $s(t) = 50t - 16t^2$. The average velocity of the ball during the first 2 seconds is

- (a) 36 ft./s
- (b) 18 ft./s
- (c) -14 ft./s
- (d) -7 ft./s

Average velocity is distance traveled divided by time elapsed, or

$$\begin{aligned}\frac{s(2) - s(0)}{2 - 0} &= \frac{[50 \cdot 2 - 16 \cdot 2^2] - [50 \cdot 0 - 16 \cdot 0^2]}{2} \\ &= \frac{100 - 64}{2} = 18.\end{aligned}$$

9. For the graph of $f(x)$ shown at right*, which of the following could be a graph of $f'(x)$?

* Please see me for an explanation of this question.

BONUS. (5 points) If $f(x) = \begin{cases} 5 & x > 3 \\ |2x - 1| & x \leq 3 \end{cases}$ then the domain of $f'(x)$ is

- (a) \mathbb{R}
- (b) $x \neq \frac{1}{2}$
- (c) $x \neq 3$
- (d) $x \neq \frac{1}{2}, x \neq 3$
- (e) none of these.

The graph of $f(x)$ consists of three straight lines as shown below. The derivative $f'(x)$ is not defined at $x = \frac{1}{2}$ and $x = 3$. Therefore the domain of $f'(x)$ is all real numbers except $\frac{1}{2}$ and 3.

