1. $\int \frac{3}{t^2}\, dt =$

   (a) $\boxed{-\frac{3}{t} + C}$

   (b) $\frac{3}{t} + C$

   (c) $-\frac{6}{t^3} + C$

   (d) $\frac{6}{t^3} + C$

   We have

$$\begin{aligned}\int \frac{3}{t^2}\, dt &= \int 3t^{-2}\, dt \\ &= \frac{3}{-1t^{-1}} + C \\ &= -\frac{3}{t} + C.\end{aligned}$$

2. The area under the graph of $f(x) = \sqrt[3]{x}$ from $x = 2$ to $x = 5$ is

   (a) $\lim_{n\to\infty}\left(\sum_{i=1}^{n} \sqrt[3]{x_i} \cdot \frac{2}{n}\right)$

   (b) $\boxed{\lim_{n\to\infty}\left(\sum_{i=1}^{n} \sqrt[3]{x_i} \cdot \frac{3}{n}\right)}$

   (c) $\lim_{n\to\infty}\left(\sum_{i=1}^{n} \sqrt[3]{x_i} \cdot \frac{5}{n}\right)$

   (d) $\lim_{n\to\infty}\left(\sum_{i=1}^{n} \sqrt[3]{x_i}\right)$

   The formula for the area under $f(x)$ from $x = a$ to $x = b$ is

$$\lim_{n\to\infty}\left(\sum_{i=1}^{n} \sqrt[3]{x_i}\Delta x\right),$$

   where $\Delta x = \frac{b-a}{n}$. In this case $\Delta x = \frac{5-2}{n} = \frac{3}{n}$, so the formula is $\lim_{n\to\infty}\left(\sum_{i=1}^{n} \sqrt[3]{x_i} \cdot \frac{3}{n}\right)$.

3. $\displaystyle\int_0^4 |x-2|\,dx =$

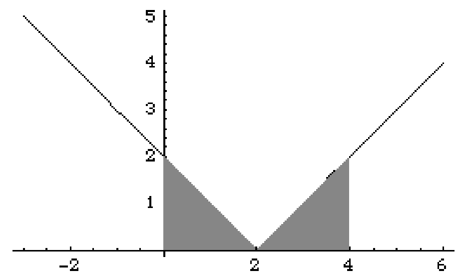
(a) 2

(b) $\boxed{4}$

(c) 0

(d) does not exist.

$\displaystyle\int_0^4 |x-2|\,dx$ represents the combined area of the regions shown at right, which is 4 (each triangle has area 2).



4. $\displaystyle\int_0^{\pi/2} 5\sin\theta\,d\theta =$

(a) 10

(b) 0

(c) $\boxed{5}$

(d) -5

We have

$$\begin{aligned}\int_0^{\pi/2} 5\sin\theta\,d\theta &= -5\cos\theta\,\Big|_0^{\pi/2} \\ &= -5\left(\cos\left(\tfrac{\pi}{2}\right) - \cos 0\right) \\ &= -5(0-1) = 5.\end{aligned}$$

5. $\displaystyle\int_0^1 x^2(1+x^3)^4\,dx =$

(a) $\frac{31}{5}$

(b) $\boxed{\frac{31}{15}}$

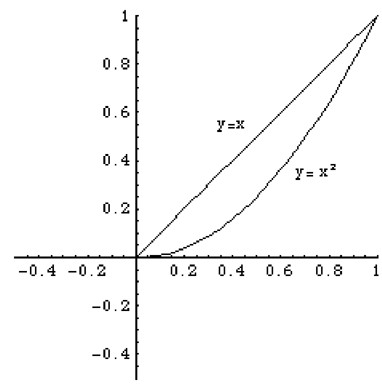
(c) $\frac{32}{5}$

(d) $\frac{32}{15}$

Let $u = 1 + x^3$. Then $du = 3x^2\,dx$, and we get

$$\begin{aligned}\int_0^1 x^2(1+x^3)^4\,dx &= \frac{1}{3}\int_0^1 3x^2(1+x^3)^4\,dx \\ &= \frac{1}{3}\int_?^? u^4\,du = \frac{1}{3}\cdot\frac{u^5}{5}\Big|_?^? \\ &= \frac{1}{15}(1+x^3)^5\Big|_0^1 \\ &= \frac{1}{15}\left((1+1^3)^5-(1+0^3)^5\right) \\ &= \frac{1}{15}(32-1) = \frac{31}{15}.\end{aligned}$$

6. The area of the region shown is



(a) $\frac{1}{9}$
(b) $\frac{1}{4}$
(c) $\frac{1}{7}$
(d) $\boxed{\frac{1}{6}}$

Since $y = x$ is the curve on top and $y = x^2$ is the curve on the bottom, the area is

$$\begin{aligned}\int_0^1 (x - x^2)\,dx &= \frac{x^2}{2} - \frac{x^3}{3}\Big|_0^1 \\ &= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}.\end{aligned}$$

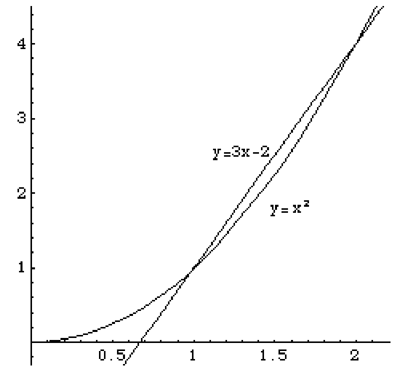
7. The volume of the solid formed by rotating the region enclosed by the curves $y = x^2$ and $y = 3x - 2$ about the x-axis is

(a) $2\pi\displaystyle\int_1^2 x(x^2 - (3x-2))\,dx$

(b) $2\pi\displaystyle\int_1^2 x(3x - 2 - x^2)\,dx$

(c) $\pi\displaystyle\int_1^2 (x^4 - (3x-2)^2)\,dx$

(d) $\boxed{\pi\displaystyle\int_1^2 ((3x-2)^2 - x^4)\,dx}$



The region is shown above. From the answer choices you can see that the curves intersect at $x = 1$ and $x = 2$ (or you can set $x^2 = 3x - 2$ and solve for x). Since we are rotating a region formed by <u>functions of x</u> about a *horizontal* axis, we should use the **disk** method. Therefore we have $R = 3x - 2$ and $r = x^2$, and the volume is

$$V = \pi \int_1^2 \left((3x-2)^2 - (x^2)^2\right)\,dx.$$

8. The volume of the solid formed by rotating the region enclosed by the curves $y = x^2$ and $y = 3x - 2$ about the y-axis is

   (a) π

   (b) $\boxed{\frac{\pi}{2}}$

   (c) $\pi\left(7 - \frac{33}{5}\right)$

   (d) $\pi\left(15 - \frac{33}{5}\right)$

This is the same region as in #7, but this time we are rotating about a *vertical* axis. So we use the **shell** method, and the volume is

$$\begin{aligned} V &= 2\pi \int_1^2 x(3x - 2 - x^2)\,dx \\ &= 2\pi \int_1^2 (3x^2 - 2x - x^3)\,dx \\ &= 2\pi\left(x^3 - x^2 - \tfrac{1}{4}x^4\right)\Big|_1^2 \\ &= 2\pi\left((8-4-4) - \left(1 - 1 - \tfrac{1}{4}\right)\right) \\ &= 2\pi\left(0 - \left(-\tfrac{1}{4}\right)\right) \\ &= \frac{\pi}{2}. \end{aligned}$$

9. If 24 lbs. of force are required to stretch a spring 18 in. (= 1.5 ft.) beyond its natural length, which expression best represents the work done in stretching the spring 10 ft. beyond its natural length? *Hint.* Remember Hooke's Law: $F(x) = kx$.

(a) $\displaystyle\int_0^{10} \frac{4}{3}x\, dx$

(b) $\displaystyle\int_0^{1.5} 10x\, dx$

(c) $\displaystyle\int_0^{1.5} 8x\, dx$

(d) $\boxed{\displaystyle\int_0^{10} 16x\, dx}$

By Hooke's Law, $24 = k \cdot 1.5$ for this spring, so the spring constant is $k = \frac{24}{1.5} = \frac{48}{3} = 16$. Therefore the force on the spring when stretched x units beyond its natural length is $F(x) = 16x$, and the work done to stretch it 10 ft. is $\displaystyle\int_0^{10} 16x\, dx$.

10. The average value of the function $f(x) = \sin\left(\frac{\pi}{2}x\right)$ on the interval $[0, 2]$ is

(a) $\frac{4}{\pi}$

(b) $\frac{3}{\pi}$

(c) $\boxed{\frac{2}{\pi}}$

(d) $\frac{1}{\pi}$

The average value of $f(x) = \sin\left(\frac{\pi}{2}x\right)$ on the interval $[0, 2]$ is

$$\frac{1}{2}\int_0^2 \sin\left(\frac{\pi}{2}x\right)\, dx = \frac{1}{2}\cdot\frac{2}{\pi}\left(-\cos\left(\frac{\pi}{2}x\right)\right)\Bigg|_0^2$$

use a u-substitution with $u = \frac{\pi}{2}x$ as in the practice midterm, #13.

$$\begin{aligned} &= -\frac{1}{\pi}\left(\cos\pi - \cos 0\right) \\ &= -\frac{1}{\pi}(-1-1) = \frac{2}{\pi}. \end{aligned}$$

BONUS. (5 points) The volume of the solid formed by rotating the rectangle with vertices $(2,2)$, $(3,3)$, $(5,1)$, and $(4,0)$ about the line $y=-x+2$ is

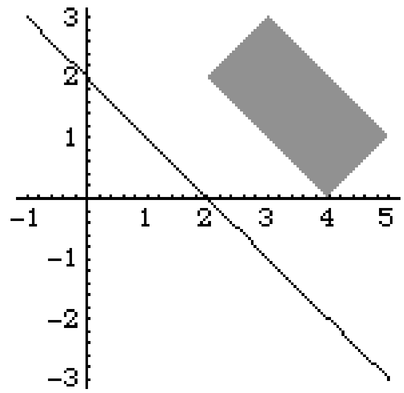
(a) 8π

(b) 4π

(c) $\boxed{12\pi\sqrt{2}}$

(d) $8\pi\sqrt{2}$

(e) $4\pi\sqrt{2}$



The rectangle is shown along with the line $y=-x+2$. When rotated about the line, the rectangle makes a cylinder with inner radius $r=\sqrt{2}$, outer radius $R=2\sqrt{2}$, and height $h=2\sqrt{2}$. Therefore the volume is

$$\begin{aligned} V &= \pi(R^2-r^2)h \\ &= \pi((2\sqrt{2})^2-(\sqrt{2})^2)2\sqrt{2} \\ &= \pi(8-2)2\sqrt{2} = 12\sqrt{2}. \end{aligned}$$