

1. $\int \frac{3}{t^2} dt =$

(a) $\frac{6}{t^3} + C$

(b) $-\frac{6}{t^3} + C$

(c) $\frac{3}{t} + C$

(d) $\boxed{-\frac{3}{t} + C}$

We have

$$\begin{aligned} \int \frac{3}{t^2} dt &= \int 3t^{-2} dt \\ &= \frac{3}{-1t^{-1}} + C \\ &= -\frac{3}{t} + C. \end{aligned}$$

2. The area under the graph of $f(x) = \sqrt[3]{x}$ from $x = 2$ to $x = 5$ is

(a) $\lim_{n \rightarrow \infty} \left(\sum_{i=1}^n \sqrt[3]{x_i} \right)$

(b) $\lim_{n \rightarrow \infty} \left(\sum_{i=1}^n \sqrt[3]{x_i} \cdot \frac{2}{n} \right)$

(c) $\lim_{n \rightarrow \infty} \left(\sum_{i=1}^n \sqrt[3]{x_i} \cdot \frac{5}{n} \right)$

(d) $\boxed{\lim_{n \rightarrow \infty} \left(\sum_{i=1}^n \sqrt[3]{x_i} \cdot \frac{3}{n} \right)}$

The formula for the area under $f(x)$ from $x = a$ to $x = b$ is

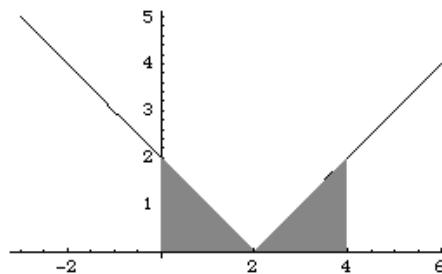
$$\lim_{n \rightarrow \infty} \left(\sum_{i=1}^n \sqrt[3]{x_i} \Delta x \right),$$

where $\Delta x = \frac{b-a}{n}$. In this case $\Delta x = \frac{5-2}{n} = \frac{3}{n}$, so the formula is $\lim_{n \rightarrow \infty} \left(\sum_{i=1}^n \sqrt[3]{x_i} \cdot \frac{3}{n} \right)$.

3. $\int_0^4 |x - 2| dx =$

- (a) 4
- (b) 2
- (c) 0
- (d) does not exist.

$\int_0^4 |x - 2| dx$ represents the combined area of the regions shown at right, which is 4 (each triangle has area 2).



4. $\int_0^{\pi/2} 5 \sin \theta d\theta =$

- (a) 5
- (b) -5
- (c) 10
- (d) 0

We have

$$\begin{aligned} \int_0^{\pi/2} 5 \sin \theta d\theta &= -5 \cos \theta \Big|_0^{\pi/2} \\ &= -5 \left(\cos \left(\frac{\pi}{2} \right) - \cos 0 \right) \\ &= -5(0 - 1) = 5. \end{aligned}$$

5. $\int_0^1 x^2(1 + x^3)^4 dx =$

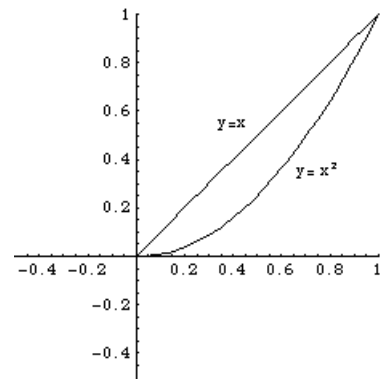
- (a) $\frac{31}{15}$
- (b) $\frac{32}{15}$
- (c) $\frac{31}{5}$
- (d) $\frac{32}{5}$

Let $u = 1 + x^3$. Then $du = 3x^2 dx$, and we get

$$\begin{aligned} \int_0^1 x^2(1+x^3)^4 dx &= \frac{1}{3} \int_0^1 3x^2(1+x^3)^4 dx \\ &= \frac{1}{3} \int_{?}^{?} u^4 du = \frac{1}{3} \cdot \frac{u^5}{5} \Big|_{?}^{?} \\ &= \frac{1}{15} (1+x^3)^5 \Big|_0^1 \\ &= \frac{1}{15} ((1+1^3)^5 - (1+0^3)^5) \\ &= \frac{1}{15} (32 - 1) = \frac{31}{15}. \end{aligned}$$

6. The area of the region shown is

- (a) $\frac{1}{9}$
- (b) $\frac{1}{4}$
- (c) $\frac{1}{6}$
- (d) $\frac{1}{7}$

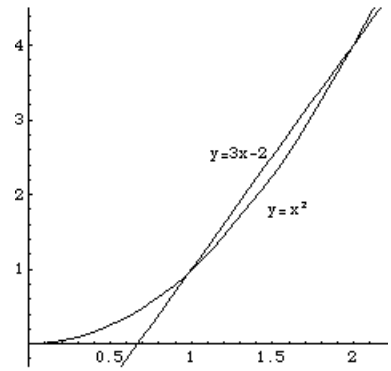


Since $y = x$ is the curve on top and $y = x^2$ is the curve on the bottom, the area is

$$\begin{aligned} \int_0^1 (x - x^2) dx &= \frac{x^2}{2} - \frac{x^3}{3} \Big|_0^1 \\ &= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}. \end{aligned}$$

7. The volume of the solid formed by rotating the region enclosed by the curves $y = x^2$ and $y = 3x - 2$ about the x -axis is

- (a) $\pi \int_1^2 (x^4 - (3x - 2)^2) dx$
- (b) $\pi \int_1^2 ((3x - 2)^2 - x^4) dx$
- (c) $2\pi \int_1^2 x(x^2 - (3x - 2)) dx$
- (d) $2\pi \int_1^2 x(3x - 2 - x^2) dx$



The region is shown above. From the answer choices you can see that the curves intersect at $x = 1$ and $x = 2$ (or you can set $x^2 = 3x - 2$ and solve for x). Since we are rotating a region formed by functions of x about a *horizontal* axis, we should use the **disk** method. Therefore we have $R = 3x - 2$ and $r = x^2$, and the volume is

$$V = \pi \int_1^2 ((3x - 2)^2 - (x^2)^2) dx.$$

8. The volume of the solid formed by rotating the region enclosed by the curves $y = x^2$ and $y = 3x - 2$ about the y -axis is

- (a) $\pi \left(7 - \frac{33}{5}\right)$
- (b) $\pi \left(15 - \frac{33}{5}\right)$
- (c) $\boxed{\frac{\pi}{2}}$
- (d) π

This is the same region as in #7, but this time we are rotating about a *vertical* axis. So we use the **shell** method, and the volume is

$$\begin{aligned} V &= 2\pi \int_1^2 x(3x - 2 - x^2) dx \\ &= 2\pi \int_1^2 (3x^2 - 2x - x^3) dx \\ &= 2\pi \left(x^3 - x^2 - \frac{1}{4}x^4\right) \Big|_1^2 \\ &= 2\pi \left((8 - 4 - 4) - \left(1 - 1 - \frac{1}{4}\right)\right) \\ &= 2\pi \left(0 - \left(-\frac{1}{4}\right)\right) \\ &= \frac{\pi}{2}. \end{aligned}$$

9. If 24 lbs. of force are required to stretch a spring 18 in. (= 1.5 ft.) beyond its natural length, which expression best represents the work done in stretching the spring 10 ft. beyond its natural length? *Hint.* Remember Hooke's Law: $F(x) = kx$.

(a) $\int_0^{10} \frac{4}{3}x \, dx$

(b) $\int_0^{10} 16x \, dx$

(c) $\int_0^{1.5} 10x \, dx$

(d) $\int_0^{1.5} 8x \, dx$

By Hooke's Law, $24 = k \cdot 1.5$ for this spring, so the spring constant is $k = \frac{24}{1.5} = \frac{48}{3} = 16$. Therefore the force on the spring when stretched x units beyond its natural length is $F(x) = 16x$, and the work done to stretch it 10 ft. is $\int_0^{10} 16x \, dx$.

10. The average value of the function $f(x) = \sin\left(\frac{\pi}{2}x\right)$ on the interval $[0, 2]$ is

(a) $\frac{3}{\pi}$

(b) $\frac{4}{\pi}$

(c) $\frac{2}{\pi}$

(d) $\frac{1}{\pi}$

The average value of $f(x) = \sin\left(\frac{\pi}{2}x\right)$ on the interval $[0, 2]$ is

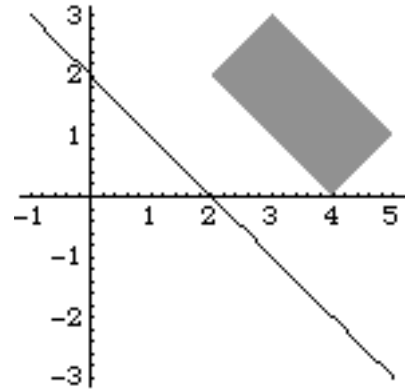
$$\frac{1}{2} \int_0^2 \sin\left(\frac{\pi}{2}x\right) \, dx = \frac{1}{2} \cdot \frac{2}{\pi} \left(-\cos\left(\frac{\pi}{2}x\right)\right) \Big|_0^2$$

use a u -substitution with $u = \frac{\pi}{2}x$ as in the practice midterm, #13.

$$\begin{aligned} &= -\frac{1}{\pi} (\cos \pi - \cos 0) \\ &= -\frac{1}{\pi} (-1 - 1) = \frac{2}{\pi}. \end{aligned}$$

BONUS. (5 points) The volume of the solid formed by rotating the rectangle with vertices $(2, 2)$, $(3, 3)$, $(5, 1)$, and $(4, 0)$ about the line $y = -x + 2$ is

- (a) 8π
- (b) 4π
- (c) $8\pi\sqrt{2}$
- (d) $12\pi\sqrt{2}$
- (e) $4\pi\sqrt{2}$



The rectangle is shown along with the line $y = -x + 2$. When rotated about the line, the rectangle makes a cylinder with inner radius $r = \sqrt{2}$, outer radius $R = 2\sqrt{2}$, and height $h = 2\sqrt{2}$. Therefore the volume is

$$\begin{aligned} V &= \pi(R^2 - r^2)h \\ &= \pi((2\sqrt{2})^2 - (\sqrt{2})^2)2\sqrt{2} \\ &= \pi(8 - 2)2\sqrt{2} = 12\sqrt{2}. \end{aligned}$$