Math 75 Quiz 2 - v. 2 (green) Solutions
Sections 4.1, 4.3, 4.4, 4.5

1. **Multiple Choice.** (5 points) *Circle the letter of the best answer.*

   The critical numbers of the function $f(x) = 2x^{1/5} - 1$ are

   (a) $\left(\frac{2}{5}\right)^{5/4}$ only

   (b) $\boxed{0 \text{ only}}$

   (c) 0 and $\left(\frac{2}{5}\right)^{5/4}$ only

   (d) There are no critical numbers of $f(x)$.

   The domain of $f(x)$ is all real numbers. The domain of $f'(x) = \frac{2}{5}x^{-5/4} = \frac{2}{5x^{5/4}}$ is $x \neq 0$. Therefore $x = 0$ is a critical number of $f(x)$. Since there is no x-value for which $f'(x) = 0$, there are no other critical numbers of $f(x)$.

2. **Multiple Choice.** (5 points) *Circle the letter of the best answer.*

   The function $g(x) = -x^2 + 6x + 1$

   (a) has an inflection point at $x = 3$

   (b) has a local minimum at $x = 3$

   (c) $\boxed{\text{has a local maximum at } x = 3}$

   (d) none of these.

   $g'(x) = -2x + 6$, which is 0 when $x = 3$. Therefore 3 is a critical number of $f(x)$.

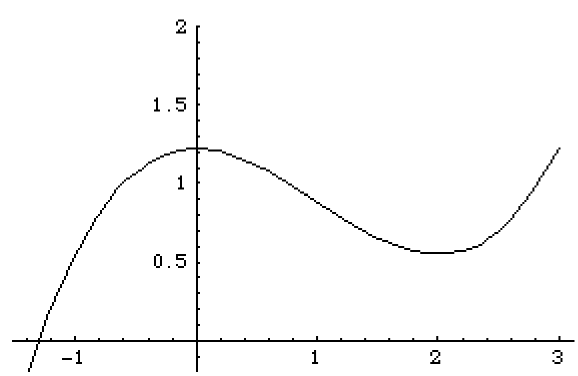
   Since $g(x)$ is a parabola opening down, the critical number must be the location of a local maximum.

   Alternatively, we can test the intervals $(-\infty, 3)$ and $(3, \infty)$. Since $g'(0) = 6 > 0$, we know $g(x)$ is increasing on the interval $(-\infty, 3)$. Since $g'(4) = -2 < 0$, we know $g(x)$ is decreasing on the interval $(3, \infty)$.

3. **Graph.** (5 points) On the axes below, sketch the graph of a function $f(x)$ satisfying all of the following:

   - $f(x)$ is increasing for all $x < 0$.
   - $f(x)$ is concave down for all $x < 1$
   - $f(x)$ has an inflection point at $x = 1$
   - $f(x)$ has a local minimum at $x = 2$

   Answers will vary.



4. (10 points) Find $\lim_{x\to\infty} \dfrac{x^5 - 5x^4 + 1}{-3x^5 + 2x}$.

Show all steps!

We have

$$\begin{aligned}
\lim_{x\to\infty} \frac{x^5 - 5x^4 + 1}{-3x^5 + 2x} &= \lim_{x\to\infty} \frac{x^5 - 5x^4 + 1}{-3x^5 + 2x} \cdot \frac{\frac{1}{x^5}}{\frac{1}{x^5}} \\
&= \lim_{x\to\infty} \frac{1 - \frac{5}{x} + \frac{1}{x^5}}{-3 + \frac{2}{x^4}} \\
&= \frac{1 - 0 + 0}{-3 + 0} \\
&= \boxed{-\frac{1}{3}}
\end{aligned}$$

BONUS. (2 points) If the derivative of $f(x)$ is $f'(x) = \dfrac{x^2 - 5}{x - 3}$, find the x-coordinate(s) of the inflection point(s) of $f(x)$.

The inflection points of $f(x)$ occur at x-values for which $f''(x) = 0$. Since $f'(x) = \dfrac{x^2 - 5}{x - 3}$, we have

$$\begin{aligned}
f''(x) &= \frac{(x-3)2x - (x^2 - 5)}{(x-3)^2} \\
&= \frac{2x^2 - 6x - x^2 + 5}{(x-3)^2} \\
&= \frac{x^2 - 6x + 5}{(x-3)^2} \overset{\text{set}}{=} 0 \\
& x^2 - 6x + 5 = 0 \\
& (x-1)(x-5) = 0 \\
& \boxed{x = 1, \quad x = 5}
\end{aligned}$$