Math 75 Practice for Quiz 3 - Solutions
Sections 4.10, 5.1-5.5

1. **Multiple Choice.** *Circle the letter of the best answer.* The general antiderivative of $f(x) = \sin x - \frac{1}{x^2}$ is

   (a) $\cos x + \frac{1}{x} + C$

   (b) $\boxed{-\cos x + \frac{1}{x} + C}$

   (c) $\cos x - \frac{1}{x} + C$

   (d) $-\cos x - \frac{1}{x} + C$

   $\sin x - \frac{1}{x^2} = \sin x - x^{-2}$, so the general antiderivative is $-\cos x + x^{-1} + C = -\cos x + \frac{1}{x} + C$. To check, note that

$$\begin{aligned}\frac{d}{dx}\left(-\cos x + \frac{1}{x} + C\right) &= \frac{d}{dx}\left(-\cos x + x^{-1} + C\right)\\ &= \sin x - x^{-2}\\ &= \sin x - \frac{1}{x^2}.\end{aligned}$$

2. **Multiple Choice.** *Circle the letter of the best answer.* The area under the graph of $f(x) = \dfrac{\sin x}{x}$ is

   (a) $\displaystyle\sum_{i=1}^{n} \frac{\sin x_i}{x_i}\Delta x$

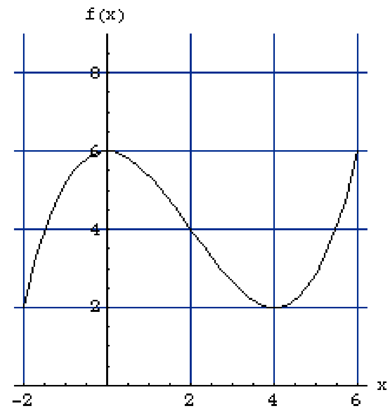
   (b) $\displaystyle\sum_{i=1}^{n} \frac{x_i \cos x_i - \sin x_i}{x_i^2}\Delta x$

   (c) $\boxed{\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n} \frac{\sin x_i}{x_i}\Delta x}$

   (d) $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n} \frac{x_i \cos x_i - \sin x_i}{x_i^2}\Delta x$

   The formula is $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$, and the function is $f(x) = \dfrac{\sin x}{x}$. So $f(x_i) = \dfrac{\sin x_i}{x_i}$, and therefore the area is $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n} \frac{\sin x_i}{x_i}\Delta x$.

3. Estimate the area under the graph of $f(x)$ from $x = -2$ to $x = 6$ with 4 rectangles using right endpoints.



The interval $[-2, 6]$ is 8 units wide, and we are using 4 rectangles. Therefore each rectangle is 2 units wide. The right endpoints of the bases of the 4 rectangles are 0, 2, 4, and 6. We have $f(0) = 6$, $f(2) = 4$, $f(4) = 2$, and $f(6) = 6$. Therefore the areas of the rectangles are $6 \cdot 2$, $4 \cdot 2$, $4 \cdot 2$, and $6 \cdot 2$. Since $f(x)$ is above the x-axis on all of $[-2, 6]$, we add these areas to get the estimate:

$$\text{Area } \approx 6 \cdot 2 + 4 \cdot 2 + 2 \cdot 2 + 6 \cdot 2 = 12 + 8 + 4 + 12 = 36.$$

4. Evaluate $\int_0^3 \sqrt{9 - x^2}\, dx$ by interpreting it in terms of areas.

The graph of $y = \sqrt{9 - x^2}$ is the upper half of a circle of radius 3 centered at $(0, 0)$. Therefore $\int_0^3 \sqrt{9 - x^2}\, dx$ represents the area of half of this half, or one quarter of the circle. Therefore the answer is

$$\int_0^3 \sqrt{9 - x^2}\, dx = \frac{1}{4}\pi \cdot 3^2 = \frac{9\pi}{4}.$$

5. Evaluate $\int_1^3 \frac{1}{x - 2}\, dx$.

This integral is undefined, because $\frac{1}{x-2}$ is not defined at $x = 2$ (a number in the interval $[1, 3]$).

6. Evaluate $\displaystyle\int \frac{x^5}{(x^6-2)^3}\,dx$.

$$\text{Let } u = x^6 - 2.$$
$$\text{Then } du = 6x^5\,dx.$$

Using the "futzing the constant" method, we have

$$\begin{aligned}
\int \frac{x^5}{(x^6-2)^3}\,dx &= \frac{1}{6}\int \frac{6x^5}{(x^6-2)^3}\,dx \\
&= \frac{1}{6}\int \frac{1}{u^3}\,du \\
&= \frac{1}{6}\int u^{-3}\,du \\
&= \frac{1}{6}\cdot\frac{u^{-2}}{-2} + C \\
&= -\frac{1}{12u^2} + C \\
&= -\frac{1}{12(x^6-2)^2} + C.
\end{aligned}$$

Checking, we get

$$\begin{aligned}
\frac{d}{dx}\left(-\frac{1}{12(x^6-2)^2} + C\right) &= \frac{d}{dx}\left(-\frac{1}{12}(x^6-2)^{-2} + C\right) \\
&= -\frac{1}{12}(-2)(x^6-2)^{-3}(6x^5) \\
&= \frac{2\cdot 6x^5}{12(x^6-2)^3} \\
&= \frac{x^5}{(x^6-2)^3}.
\end{aligned}$$