Math 75 Worksheet 2 Solutions
Chapter 2

Part 1. Find the following limits, if they exist. If a limit does not exist, explain why.

1. $\lim_{x\to 1}\left(3x^2 - \frac{1}{x+1}\right)$

$$\begin{aligned}\lim_{x\to 1}\left(3x^2 - \frac{1}{x+1}\right) &= 3\cdot 1^2 - \frac{1}{1+1}\\ &= 3 - \frac{1}{2} = \boxed{\frac{5}{2}}\end{aligned}$$

using the limit laws, since $3x^2 - \frac{1}{x+1}$ is continuous at $x = 1$.

2. $\lim_{x\to 2^-}\frac{x+3}{x-2}$

There is a vertical asymptote for the graph of $f(x) = \frac{x+3}{x-2}$ at $x = 2$. As we approach 2 from the left, the values of $f(x)$ become increasingly negative. Therefore

$$\lim_{x\to 2^-}\frac{x+3}{x-2} = \boxed{-\infty}.$$

3. $\lim_{h\to 0}\frac{2(3+h)^2 - 18}{h}$

$$\begin{aligned}\lim_{h\to 0}\frac{2(3+h)^2 - 18}{h} &= \lim_{h\to 0}\frac{18 + 12h + 2h^2 - 18}{h}\\ &= \lim_{h\to 0}\frac{12h + 2h^2}{h}\\ &= \lim_{h\to 0}\frac{h(12+2h)}{h}\\ &= \lim_{h\to 0} 12 + 2h = \boxed{12}.\end{aligned}$$

4. $\lim_{h\to 0}\frac{3(a+h)^2 - 3a^2}{h}$

$$\begin{aligned}\lim_{h\to 0}\frac{3(a+h)^2 - 3a^2}{h} &= \lim_{h\to 0}\frac{3a^2 + 6ah + 3h^2 - 3a^2}{h}\\ &= \lim_{h\to 0}\frac{6ah + 3h^2}{h}\\ &= \lim_{h\to 0}(6a + 3h) = \boxed{6a}.\end{aligned}$$

5. $\lim_{h \to 0} \frac{(a+h)^2 + (a+h) - (a^2+a)}{h}$

$$\begin{aligned}\lim_{h \to 0} \frac{(a+h)^2 + (a+h) - (a^2+a)}{h} &= \lim_{h \to 0} \frac{a^2 + 2ah + h^2 + a + h - a^2 - a}{h} \\ &= \lim_{h \to 0} \frac{2ah + h^2 + h}{h} \\ &= \lim_{h \to 0} (2a + h + 1) = \boxed{2a+1}.\end{aligned}$$

6. $\lim_{x \to 2} \frac{|x+1|}{x-2}$

7. $\lim_{x \to -2^+} \frac{|3x+6|}{x+2}$

Part 2. State whether or not each function $f(x)$ is continuous at the given values.

1. $f(x) = \frac{5}{(x+1)^3}$; $x = -1$

2. $f(x) = \frac{x+1}{(x+1)(x-2)}$; $x = -1$

3. $f(x) = \tan x$; $x \in \left[0, \frac{\pi}{2}\right)$

4. $f(x) = \sqrt{x}$; $x \in [0, 1]$

Part 3. More problems.

1. Determine *exactly* the slope of the tangent line to the graph of $f(x) = 2x^2$ at $x = 3$.
   Hint. See problem #3 in Part 1.

2. Find a function $f(x)$ such that the slope of the tangent line to the graph of $f(x)$ at $x = a$ is $\lim_{h \to 0} \dfrac{(a+h)^2 + (a+h) - (a^2 + a)}{h}$.

3. Sketch a graph of the function in Part 2, #1.

4. Sketch a graph of the function in Part 2, #2.