Section 6.2 - Volume - p. Stewart, 4th Ed. (lecture 2)

Recall. The Disk Method.

- The volume of a solid formed by rotating a region about the x-axis is

$$\int_a^b A(x)\, dx,$$

  where

  - the region goes from $x = a$ to $x = b$
  - the region is formed from functions of x, say $f(x)$ above and $g(x)$ below
  - $A(x) = \pi R^2 - \pi r^2$ is the cross-sectional area at x.

- If the axis of rotation is the x-axis, then $A(x) = \pi \cdot (f(x) - g(x))^2$, because the radii are just the heights of the functions at each value of x.

- If the axis of rotation is $y = c$ for $c \neq 0$, then you'll need to adjust $A(x)$ to account for this.

  The way to do it: the radius of larger circle is now $|f(x) \pm c|$ (where $f(x)$ is the "top" function). Similarly, radius of smaller circle is now $|g(x) \pm c|$. The "$\pm$" depends on whether c is positive or negative. Hence

$$A(x) = \pi\,(f(x) \pm c)^2 - \pi\,(g(x) \pm c)^2\,.$$

  Draw a picture to see whether to add or subtract.

Example 1. Find the volume of the solid obtained by rotating the region enclosed by $y = 2x$ and $y = x^2$ about the line $y = 5$.

Solution:

1. Find where the curves intersect.

   The curves intersect at $x =$ ______ and $x =$ ______.

2. Figure out what the radii are.

   Draw a picture:

Notice that if the distance from $y = x^2$ to the x-axis is x^2 and the distance from $y = 5$ to the x-axis is 5, then the difference must be $5 - x^2$. Similarly for $y = 2x$. So $R = 5 - x^2$ and $r = 5 - 2x$.

3. Find $A(x)$ and compute the integral.

   $A(x) =$

   Thus the volume is

Caution. The formula for $A(x)$ is $\pi(R^2 - r^2)$, NOT $\pi(R - r)^2$!

The disk method can also be used to find the volume of a solid formed from a region bounded by functions of y about a vertical axis; the formula is

$$\int_c^d A(y)\,dy,$$

where $A(y)$ is the cross-sectional area at y, found similarly, and c and d are y-limits.

Example 2. Find the volume of the solid obtained by rotating the region enclosed by $y^2 = x$ and $y = \frac{1}{3}x$ about the y-axis.

Solution:

1. Write the curves as functions of y.

2. Find where the curves intersect.

   Similar to the previous calculation, the curves intersect at $y =$ _____ and $y =$ _____.

3. Figure out what the radii are.

   Draw a picture:

   The radii are ______________ and ______________, so $A(y) =$ ____________________________,
   and the volume is