Section 6.5 - Average Value of a Function - p. 398 Stewart, 4th Ed.

Recall: The average of the 5 numbers 2, 8, -3, $\frac{4}{7}$, and $\sqrt{2}$ is

$$\frac{2+8+-3+\frac{4}{7}+\sqrt{2}}{5}.$$

In general, the average of n numbers y_1, y_2, y_3, ..., y_n is

$$\frac{y_1+y_2+y_3+\ldots+y_n}{n}=\frac{1}{n}\sum_{i=1}^{n} y_i.$$

How to take the average of infinitely many numbers? For example, suppose we wanted to find the average temperature for a day. We could *estimate* the average by taking several readings throughout the day and taking the average of those readings. We could get a better estimate by increasing the number of readings. Hmm! Sounds like this could turn into an integral if we let the number of readings get infinite.

Suppose the temperature is given by $f(t)$, where t is the number of hours after midnight. Let's estimate the average temperature (**average value of** $f(t)$) from $t=0$ to $t=24$ using n readings:

Estimate of average value is

$$\begin{aligned}\frac{f(t_1)+f(t_2)+\ldots+f(t_n)}{n} &= \frac{1}{n}\sum_{i=1}^{n} f(t_i)\\ &= \frac{1}{n}\cdot 1\Delta t\sum_{i=1}^{n} f(t_i)\Delta t\\ &= \frac{1}{n}\cdot n24-0\sum_{i=1}^{n} f(t_i)\Delta t\\ &= \frac{1}{24}\sum_{i=1}^{n} f(t_i)\Delta t.\end{aligned}$$

Now most of that looks like the definition of the integral if we take $\lim_{n\to\infty}$! Sure enough, that's how we get the actual average temperature: by letting the number of readings get infinite.

Average temperature for the day is

$$\frac{1}{24}\int_0^{24} f(t)\,dt.$$

In general, the **average value of** $f(x)$ from $x=a$ to $x=b$ is

$$\frac{1}{b-a}\int_a^b f(x)\,dx.$$

Example. The average value of $f(x)=\cos x$ from 0 to 2π is

This answer makes sense since $\cos x$ is "centered around" 0, and it undergoes one complete oscillation between 0 and 2π.

Example. Suppose the temperature (in degrees Fahrenheit) t hours after midnight July 24 was

$$f(t) = -\frac{1}{30}(2t^3 - 69t^2 + 612t - 3000).$$

What was the average temperature for July 24?

(You should get $f_{\text{ave}} \approx 66.4$).

Geometric interpretation. The average value of $f(x)$ is the horizontal line for which there is the same area (between the graph of $f(x)$ and the line) above the line as below. This makes sense given our interpretation of the definite integral as the area under between curves.

For example, the graphs of $f(t)$ (from the example above) and $y = 66.4$ are graphed below. Shade the areas between $f(t)$ and $y = 66.4$ on the graph.

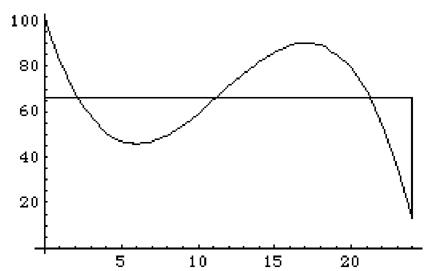

Figure 1: Comparing the graphs of $y = f(t)$ and $y = f_{\text{ave}}$

Does it seem plausible that the area you shaded above $y = 66.4$ is the same as the area you shaded below?