Math 90 Practice Midterm I Solutions
§§2-A – 3-B (Ebersole), 1.1, 1.3 (Stewart), **W1**

Multiple Choice. *Circle the letter of the best answer.*

1. A description for the function $f(x) = \sqrt{3x} + 2$ is

   (a) Take 3 times a number and then add 2
   (b) Take 3 times a number, add 2, and then take the square root of the result
   (c) $\boxed{\text{Take 3 times a number, take the square root of the result, then add 2}}$
   (d) Take $\sqrt{3}$ times a number and then add 2

   $3x$ is under the square root, so we are taking the input and multiplying it by 3, then taking the square root of the result. Finally, we add 2.

2. The range of the function $g(x) = -x^2 + 6x + 5$ is

   (a) $\mathbb{R}$ (all real numbers)
   (b) $[14, \infty)$
   (c) $[-\infty, 14)$
   (d) $\boxed{(-\infty, 14]}$

   $g(x)$ is a parabola opening down, so the range (outputs) must be from $-\infty$ to the y-coordinate of the vertex. The vertex is at $(3, 14)$. Since 14 is in the range, and $-\infty$ is not ($-\infty$ is not a real number!), the range is $(-\infty, 14]$.

3. The graph of the function $g(t) = \sqrt{9 - t^2}$ is

   (a) A circle of radius 9 centered at the origin
   (b) A circle of radius 3 centered at the origin
   (c) The upper half of a circle of radius 9 centered at the origin
   (d) $\boxed{\text{The upper half of a circle of radius 3 centered at the origin}}$

   $y = \sqrt{r^2 - t^2}$ always represents the upper half of a circle of radius r centered at the origin, since if we square both sides we get $y^2 = r^2 - t^2$, or $t^2 + y^2 = r^2$, which is the equation of a circle of radius r. We get only the upper half because $\sqrt{r^2 - t^2}$ cannot be negative for any input t.

Fill-In. If $f(x) = 3x - 5$ and $g(x) = x^3$, then

1. $(g \circ f)(1) =$ <u>-8</u>
2. $(g - f)(0) =$ <u>5</u>
3. $(f \circ f)(2) =$ <u>-2</u>

4. $(f \circ g)(-1) = \underline{\quad -8 \quad}$

First compute the formulas for $(g \circ f)(x)$, $(f \circ f)(x)$, and $(f \circ g)(x)$:

$$(g \circ f)(x) = g(f(x)) = g(3x-5) = (3x-5)^3$$
$$(f \circ f)(x) = f(f(x)) = f(3x-5) = 3(3x-5) - 5 = 9x - 15 - 5 = 9x - 20$$
$$(f \circ g)(x) = f(g(x)) = f(x^3) = 3x^3 - 5.$$

Using the first formula we get $(g \circ f)(1) = (3(1) - 5)^3 = (-2)^3 = -8$.
Using the second formula we get $(f \circ f)(2) = 9(2) - 20 = 18 - 20 = -2$.
Using the third formula we get $(f \circ g)(-1) = 3(-1)^3 - 5 = -3 - 5 = -8$.

To get $(g - f)(0)$, remember that $(g - f)(x) = g(x) - f(x)$. So

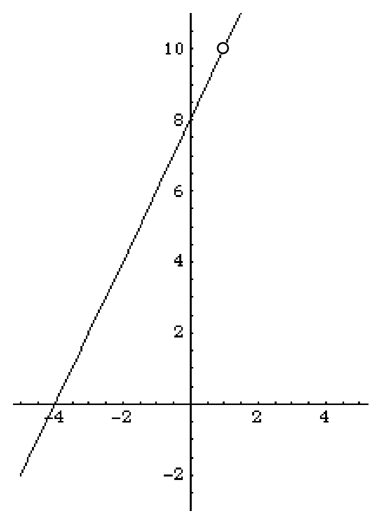
$$\begin{aligned}(g - f)(0) &= g(0) - f(0) \\ &= 0^3 - (3(0) - 5) \qquad \text{(remember the parentheses here!)} \\ &= 0 - (-5) = 5.\end{aligned}$$

Graphs.

1. On the axes below, sketch the graph of $f(x) = \dfrac{2x^2 + 6x - 8}{x - 1}$.

$$\begin{aligned} f(x) &= \frac{2x^2 + 6x - 8}{x - 1} \\ &= \frac{2(x^2 + 3x - 4)}{x - 1} \\ &= \frac{2(x-1)(x+4))}{x - 1} \\ &= 2(x+4) = 2x + 8 \end{aligned}$$

with $x \neq 1$. So the graph of $f(x)$ is identical to the graph of $y = 2x + 8$, except that there is a hole at $x = 1$. Therefore the graph looks like $\longrightarrow$



2. On the axes below, sketch the graph of $h(x) = 2|x - 1| + 3$.

There are two ways to do this problem:

(1) Transformations of $|x|$. Notice that if we perform the following transformations, we will get $h(x)$:

$$|x| \xrightarrow[\text{right 1}]{\text{shift}} |x - 1| \xrightarrow[\text{vertically}]{\text{stretch}} 2|x - 1| \xrightarrow[\text{up 3}]{\text{shift}} 2|x - 1| + 3.$$

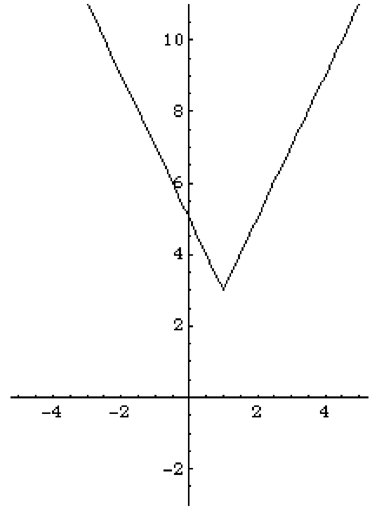
Therefore the graph looks like the picture on the next page.

(2) Piecewise function.

$$h(x) = \begin{cases} 2(x-1)+3 & \text{if } x-1 \geq 0 \\ -2(x-1)+3 & \text{if } x-1 < 0 \end{cases}$$

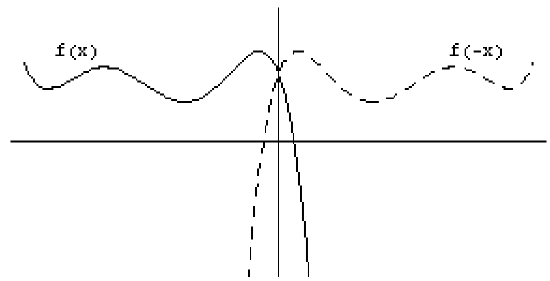
$$= \begin{cases} 2x+1 & \text{if } x \geq 1 \\ -2x+5 & \text{if } x < 1. \end{cases}$$

$h(1) = 2(1)+1 = 3$, so the vertex is at $(1,3)$, and we get the graph shown.



3. The graph of $f(x)$ is shown at right.
   On the same axes, sketch the graph of $f(-x)$.

   The graph of $f(-x)$ is shown with dashed lines. Notice that it is the **(horizontal)** reflection of $f(x)$ about the y-axis.



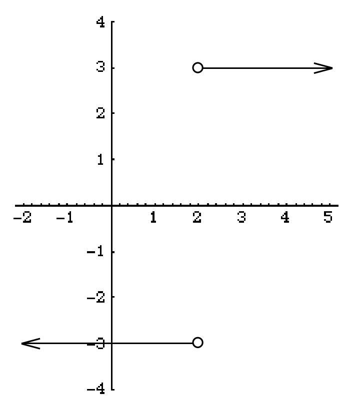
Work and Answer. *You must show all relevant work to receive full credit.*

1. Write $f(x) = \dfrac{|3x-6|}{x-2}$ as a picewise function and graph the function. What is the domain of $f(x)$?

$$f(x) = \begin{cases} \frac{3x-6}{x-2} & \text{if } 3x-6 > 0 \\ \frac{-(3x-6)}{x-2} & \text{if } 3x-6 < 0 \end{cases}$$

$$= \begin{cases} \frac{3(x-2)}{x-2} & \text{if } 3x > 6 \\ \frac{-3(x-2)}{x-2} & \text{if } 3x < 6 \end{cases}$$

$$= \begin{cases} 3 & \text{if } x > 2 \\ -3 & \text{if } x < 2. \end{cases}$$



Notice that the first case says "$>$" rather than "$\geq$" since $f(x)$ is undefined at $x = 2$. In fact, the domain of $f(x)$ is $\{x \mid x \neq 2\}$.

From the piecewise function, we can see that the graph looks like the one shown.

2. Let $f(x) = \sqrt{x+1}$.

   (a) Find the slope of the secant line to the graph of $f(x)$ from the point $(x, \sqrt{x+1})$ to the point $(a, \sqrt{a+1})$, where $x \neq a$. Simplify.

   The slope is

$$\begin{aligned}
\frac{\sqrt{x+1}-\sqrt{a+1}}{x-a} &= \frac{\sqrt{x+1}-\sqrt{a+1}}{x-a} \cdot \frac{\sqrt{x+1}+\sqrt{a+1}}{\sqrt{x+1}+\sqrt{a+1}} \\
&= \frac{(x+1)-(a+1)}{(x-a)(\sqrt{x+1}+\sqrt{a+1})} \\
&= \frac{x-a}{(x-a)(\sqrt{x+1}+\sqrt{a+1})} \\
&= \frac{1}{\sqrt{x+1}+\sqrt{a+1}}.
\end{aligned}$$

   (b) Using your answer to part (2a), find the slopes of the secant lines

      i. between the points $(1, \sqrt{2})$ and $(3, 2)$

      Here $x = 1$ and $a = 3$, so the slope is $\frac{1}{\sqrt{1+1}+\sqrt{3+1}} = \boxed{\frac{1}{\sqrt{2}+2}}$. Notice that $\sqrt{x+1} = \sqrt{2}$ and $\sqrt{a+1} = 2$, exactly the y-coordinates of the points given. So in this case we can just use the y-coordinates in the denominator.

      ii. between the points $(-1, 0)$ and $(8, 3)$

      Here $x = -1$ and $a = 8$, so the slope is $\frac{1}{0+3} = \boxed{\frac{1}{3}}$.

      iii. between the points $(-1, 0)$ and $(0, 1)$

      Here $x = -1$ and $a = 0$, so the slope is $\frac{1}{0+1} = \boxed{1}$.