Math 75B Practice Final §§3.10-6.5 (Stewart)

DISCLAIMER. This collection of practice problems is *not* guaranteed to be identical, in length or content, to the actual exam. You may expect to see problems on the test that are not exactly like problems you have seen before.

On the actual exam you will have more room to work the problems. You will see directions similar to these:

1. Please read directions carefully. Raise your hand if you are not sure what a problem is asking.
2. *You must explain your work thoroughly and unambiguously to receive full credit,* except *on questions or parts of questions designated as* ***Multiple Choice****,* ***Fill-In****, or* ***Graphs****.*
3. **No calculators or notes are allowed on this exam.**
4. You have 2 hours to complete your test, unless announced otherwise. Do not spend too long on any one problem. You do not have to do the problems in order. Do the easy ones first. Do not attempt the bonus question until you have completed the rest of the test. Before turning in your test, please make sure you have answered and double-checked all the questions.
5. If you need scratch paper, please raise your hand. You may not use your own paper. When you have finished your exam, please turn in any scratch paper you use.
6. For **Work and Answer** problems, write your solutions in the space provided for each problem, or provide specific instructions as to where your work is to be found. *Make it clear what you want and don't want graded.* Your final answers should be boxed or circled.
7. Don't stress! I'm rooting for you!

You will also see the following useful formulas:

English system formulas:	Metric system formulas:
1 ft. = 12 in.	$F = m \cdot a$
5280 ft. = 1 mi.	$g = 9.8\,\text{m/s}^2$
16 oz. = 1 lb.	100 cm = 1 m
Weight of water: $\omega = 62.5\,\text{lb./ft}^3$	Weight of water: $\omega = 9800\,\text{N/m}^3$

General formulas:

Hooke's Law: $F(x) = kx$

$$W = \omega \int_0^b (x+P)A(x)\,dx$$

Multiple Choice. *Circle the letter of the best answer.*

1. $\displaystyle\int (\sin x + 2\cos x)\,dx =$

(a) $-\cos x + 2\sin x + C$

(b) $-\cos x - 2\sin x + C$

(c) $\cos x + 2\sin x + C$

(d) $\cos x - 2\sin x$

(e) $-\cos x - 2\sin x$

2. Suppose you know that $f'(x) = g(x)$. Which of the following must be true?

(a) $\displaystyle\int g(x)\,dx = f(x)$

(b) $\displaystyle\int g(x)\,dx = f(x) + C$

(c) $\frac{d}{dx}(g(x)) = f(x)$

(d) $\frac{d}{dx}(g(x)) = f(x) + C$

(e) All of the above are true.

3. $\displaystyle\lim_{x\to-\infty} \frac{2x^5 - 3x^3 + 1}{-5x^3 + x - 1} =$

(a) $-\dfrac{2}{5}$

(b) 0

(c) ∞

(d) $-\infty$

(e) does not exist.

4. If $x < 0$, then $x^3 =$

(a) $\sqrt[4]{x^6}$

(b) $\sqrt[4]{x^8}$

(c) $-\sqrt[4]{x^8}$

(d) $\sqrt[4]{x^{12}}$

(e) $-\sqrt[4]{x^{12}}$

5. If $y = \displaystyle\int_0^{x^2} \tan t\,dt$, then $y' =$

(a) $2x\tan(x^2)$

(b) $\tan(x^2)$

(c) $\tan x$

(d) $2x\sec^2(x^2)$

(e) $\sec^2(x^2)$

6. The inflection point(s) of the function $y = 3x^5 - 5x^4 + 60x - 60$ is/are

(a) $(0, -60)$ only

(b) $(-1, -128)$ only

(c) $(-1, -128)$ and $(1, -2)$ only

(d) $(1, -2)$ only

(e) $(0, -60)$, $(1, -2)$, and $(-1, -128)$ only

7. Which of the following is the linear approximation of the function $f(x) = \sqrt[3]{x}$ near the number $a = 1$?

(a) $y = \frac{1}{3}x + 1$

(b) $y = \frac{1}{3}x + \frac{2}{3}$

(c) $y = x - \frac{2}{3}$

(d) $y = x + 3$

(e) $y = 3x + 2$

8. $\displaystyle\int_0^4 |x-3|\, dx =$

(a) 24

(b) 2

(c) 4

(d) 20

(e) 5

9. Let $\mathcal{R}$ be the region enclosed by the lines $y = 2x$, $y = 4x$, and $x = 2$. The volume of the solid formed by rotating $\mathcal{R}$ about the x-axis is

(a) $\pi \displaystyle\int_0^2 \left((2x)^2 - (4x)^2\right)\, dx$

(b) $2\pi \displaystyle\int_0^8 x(2x - 4x)\, dx$

(c) $2\pi \displaystyle\int_0^8 x(4x - 2x)\, dx$

(d) $\pi \displaystyle\int_0^2 \left((4x)^2 - (2x)^2\right)\, dx$

(e) $\pi \displaystyle\int_0^2 (4x - 2x)^2\, dx$

Fill-In.

1. $\displaystyle\lim_{x\to\infty} \frac{2x^3 - 3x^2 + 1}{4x^3 + 2x^2 + x - 1} =$ ____________ .

2. The vertical asymptotes for the function $f(x) = \frac{x}{x^2-1}$ is/are ________________________ and the horizontal asymptotes is/are ________________________ .

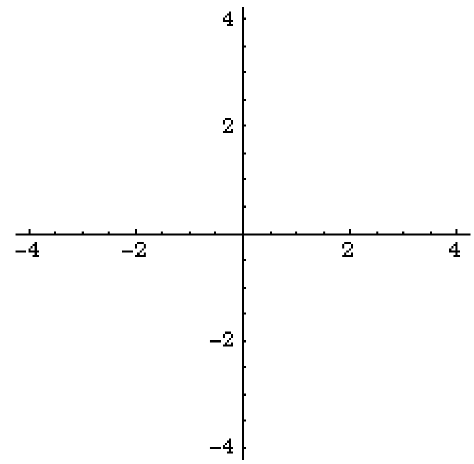
3. The graph of the function $f(x) = x^4 + 2x^3$ is increasing on the interval(s) ____________ ________________________ .

4. According to Rolle's Theorem, the maximum number of real roots of the function $f(x) = 4x^5 + 2x - 3$ is _________ .

5. Given the initial guess $x_1 = 2$, the second approximation to a root of $g(x) = x^3 - 4x - 1$ using Newton's Method is $x_2 =$ __________ .

Graphs. *More accuracy = more points!*

1. For the function $f(x) = \frac{1}{3}x^3 - 2x$,



(a) find the critical **points** and intervals of increase/decrease

(b) find the inflection **points** and intervals of concave up/concave down

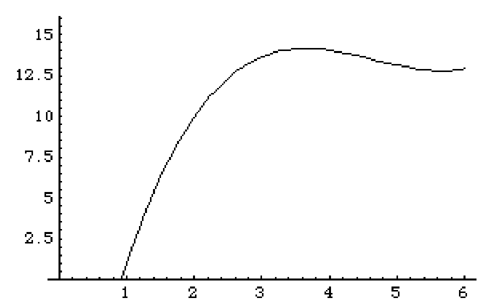
(c) discuss any symmetry $f(x)$ may or may not have

(d) find the equations of any vertical and/or horizontal asymptotes

(e) find the y-intercept

(f) On the axes at right, sketch an accurate graph of $f(x)$.

2. (a) For the function $f(x)$ graphed at right, sketch a **rectangle** on the same axes whose area is approximately $\int_1^5 f(x)\,dx$.



(b) The average value f_{ave} of $f(x)$ from $x = 1$ to $x = 5$ is approximately __________ .

(c) The approximate value(s) of c so that $f(c) = f_{\text{ave}}$ is/are ____________________ *(list all values)*.

Work and Answer. *You must show all relevant work to receive full credit.*

1. Evaluate $\int_{-1}^{2} (x^2 + 2)\,dx$.

2. Evaluate $\int x(3x^2+1)^5\,dx$.

3. A farmer has 400 meters of fencing with which to fence 3 sides of a rectangular horse corral. What is the maximum area she can enclose?

4. Find the area of the region enclosed by the curves $y = 4 - x^2$ and $y = x + 2$.

5. Evaluate $\int_0^1 x\cos(x^2+1)\,dx$.

6. Let $\mathcal{R}$ be the region enclosed by the graphs of $y = x^2$, the x-axis, and $x = 3$. Find the volume of the solid formed by rotating $\mathcal{R}$ about the y-axis.

7. If 9 J of work are required to stretch a spring 75 cm beyond its natural length, find the work done in stretching the spring from 75 cm to 1 m beyond its natural length.

Some kind of **BONUS.**