**Math 75B Practice Midterm I**
Ch. 12, 16, 17 (Ebersole), §§3.10-4.9 (Stewart)

**DISCLAIMER.** This collection of practice problems is *not* guaranteed to be identical, in length or content, to the actual exam. You may expect to see problems on the test that are not exactly like problems you have seen before.

On the actual exam you will have more room to work the problems. You will see directions similar to these:

1. Please read directions carefully. Raise your hand if you are not sure what a problem is asking.
2. *You must explain your work thoroughly and unambiguously to receive full credit,* except *on questions or parts of questions designated as* ***True or False****,* ***Multiple Choice****,* ***Fill-In****, or* ***Graph****.*
3. **No calculators or notes are allowed on this exam.**
4. You have 50 minutes to complete your test, unless announced otherwise. Do not spend too long on any one problem. You do not have to do the problems in order. Do the easy ones first. Do not attempt the bonus question until you have completed the rest of the test. Before turning in your test, please make sure you have answered and double-checked all the questions.
5. If you need scratch paper, please raise your hand. You may not use your own paper. When you have finished your exam, please turn in any scratch paper you use.
6. Write your solutions in the space provided for each problem, or provide specific instructions as to where your work is to be found. *Make it clear what you want and don't want graded.* Your final answers should be boxed or circled.
7. Don't stress! I'm rooting for you!

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**True or False.** Circle **T** if the statement is *always* true; otherwise circle **F**.

1. If $f(x)$ is a continuous function and $f(3) = 2$ and $f(5) = -1$, then $f(x)$ has a root between 3 and 5. **T** **F**
2. The function $g(x) = 2x^3 - 12x + 5$ has 5 real roots. **T** **F**
3. If $h(x)$ is a continuous function and $f(1) = 4$ and $f(2) = 5$, then $f(x)$ has no roots between 1 and 2. **T** **F**
4. The only $x$-intercept of $f(x) = x^3 - x^2 + 2x - 2$ is $(1, 0)$. *(Challenge problem!)* **T** **F**

**Multiple Choice.** *Circle the letter of the best answer.*

1. The absolute minimum of $f(x) = -x^2 + 6x + 1$ on the interval $[0, 5]$ is at $x =$

   (a) 0
   (b) 1
   (c) 2
   (d) 3

2. The function $f(x) = \cos x - x$

   (a) is an even function
   (b) is an odd function
   (c) is neither an even nor an odd function

3. The function $f(x) = x^4 - 6x^2$ is increasing on the intervals

   (a) $(0, \sqrt{3})$ only
   (b) $(-\infty, -\sqrt{3})$ and $(0, \sqrt{3})$ only
   (c) $(\sqrt{3}, \infty)$ only
   (d) $(-\sqrt{3}, 0)$ and $(\sqrt{3}, \infty)$ only

4. The function $f(x) = x^4 - 6x^2$ is concave down on the intervals

   (a) $(-1, 1)$ only
   (b) $(-\sqrt{3}, \sqrt{3})$ only
   (c) $(-\infty, -1)$ and $(1, \infty)$ only
   (d) $(1, \sqrt{3})$ only

5. The linear approximation of $f(x) = \sqrt{5 - x}$ at $x = 1$ is

   (a) $y = -\frac{1}{4}x + \frac{9}{4}$
   (b) $y = -\frac{3}{4}x + \frac{7}{4}$
   (c) $y = \frac{1}{4}x + \frac{7}{4}$
   (d) $y = -\frac{3}{4}x + \frac{9}{4}$

6. If $x_1 = 1$ is a first approximation of a solution to the equation $x^4 = 6 - 3x$, then using Newton's Method the second approximation is $x_2 =$

   (a) $\frac{9}{7}$
   (b) $\frac{5}{7}$
   (c) $\frac{9}{2}$
   (d) $-\frac{5}{2}$

**Fill-In.**

1. The horizontal asymptote(s) of the function $f(x) = \dfrac{\sqrt{2x^6 - 1}}{x^3 + 2x^2 + 5}$ is/are ______________ .

2. Using a tangent line approximation, $\sqrt[3]{126.5} \approx$ ____________.

3. The absolute maximum value of the function $g(x) = \frac{3}{x-5}$ on the interval $[-3, -1]$ is ____________ .

4. If a polynomial function $f(x)$ has 3 solutions to the equation $f'(x) = 0$, then $f(x)$ has at most _______ roots.

5. A contractor has 80 ft. of fencing with which to build three sides of a rectangular enclosure. In order to enclose the largest possible area, the dimensions of the enclosure should be ______ $\times$ ______.

**Work and Answer.** *You must show all relevant work to receive full credit.*

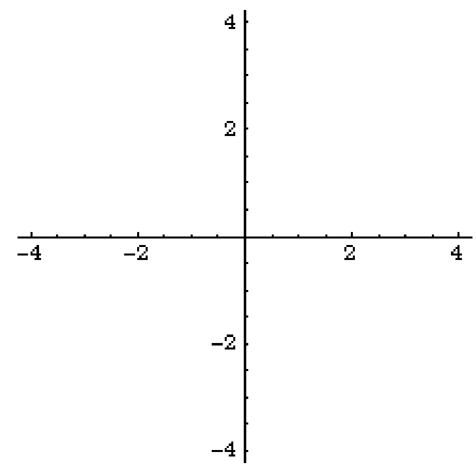
1. A cone-shaped roof with base radius $r = 6$ ft. is to be covered with a 0.5-inch layer of tar. Use differentials to estimate the amount of tar required (you may use the formula $V(r) = \frac{2}{9}\pi r^3$ for the volume of the piece of the house covered by the roof).

2. If 1200 $\text{cm}^2$ of sheet metal is available to make a box with a square base and open top, find the largest possible volume of the box.

3. Last month I drove to my friend's house 150 miles away. The trip took 3 hours. Explain why there was at least one moment during the trip at which I was driving exactly 49 miles per hour.

4. Estimate the root of $f(x) = x^3 + 2x - 1$ using two iterations of Newton's Method (*i.e.* compute $x_3$) with the initial guess $x_1 = 0$. *Express your answer as an exact fraction.*

5. For the function $g(x) = \frac{2}{3}x^3 - 2x^2$,




(a) find the critical **points** and intervals of increase/decrease

(b) find the inflection **points** and intervals of concave up/concave down

(c) discuss any symmetry $g(x)$ may or may not have

(d) find the equations of any vertical and/or horizontal asymptotes

(e) find the $y$-intercept

(f) On the axes at right, sketch an accurate graph of $g(x)$.

6. Prove that the function $f(x) = -x^3 - 6x + 1$ has exactly one real root by completing the following:

   (a) Use the Intermediate Value Theorem to show that the function $f(x) = -x^3 - 6x + 1$ has *at least* one real root.

   (b) Use Rolle's Theorem to show that the function $f(x) = -x^3 - 6x + 1$ has *at most* one real root.

Some kind of **BONUS.**