d. $(f \circ g)(x)$ is only defined when $g(x)$ is defined and $f(g(x)) = \frac{3}{x-1} + 2$ is defined; this is true for all $x \neq 1$. The domain of $f \circ g$ is the union of the intervals $(-\infty,1) \cup (1,\infty)$.

e. $(g \circ f)(x)$ is only defined when $f(x)$ is defined and $g(f(x)) = \frac{3}{x+1}$ is defined. This is true for all $x \neq -1$. The domain of $g \circ f$ is the union of the intervals $(-\infty, -1) \cup (-1, \infty)$. ■

EXAMPLE 3.7 Consider the graphs of $y = f(x)$ and $y = g(x)$ shown in Figures 3.6 and 3.7 (which we repeat below). Estimate the values of the composite functions indicated in parts a, b, c, and d if they are defined. If they are undefined, explain why.

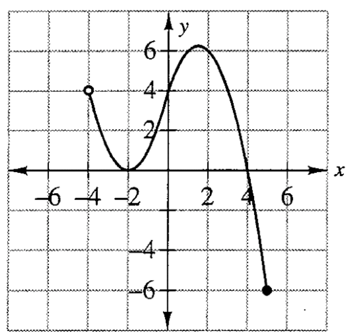

Figure 3.6 $y = f(x)$

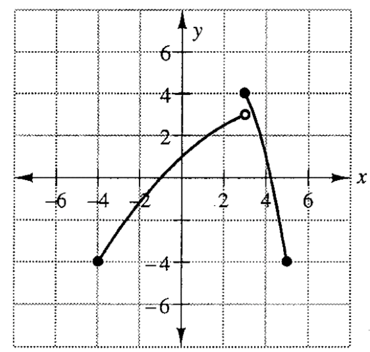

Figure 3.7 $y = g(x)$

a. $(g \circ f)(3)$ **b.** $(f \circ g)(3)$ **c.** $(f \circ f)(5)$ **d.** $(g \circ g)(-1)$

Solution

a. Since $f(3) = 4$ and $g(4) = 1$, $(g \circ f)(3) = g(f(3)) = g(4) = 1$.

b. $(f \circ g)(3) = f(g(3)) = f(4) = 0$.

c. $(f \circ f)(5) = f(f(5)) = f(-6)$. But -6 is not in the domain of f (there is no point on the graph of f with x-coordinate -6), so $f(-6)$ is not defined. This means that $(f \circ f)(5)$ is not defined.

d. $(g \circ g)(-1) = g(g(-1)) = g(0) = 1$. ■

Exercises 3-A

1. Use the graphs given in Figures 3.6 and 3.7 to estimate the following values if they are defined. If they are undefined, explain why.

a. $(f \circ g)(-1)$ **b.** $\left(\frac{g}{f}\right)(-1)$ **c.** $(fg)(4)$ **d.** $(g - f)(0)$