Worksheet - Techniques of Integration Necessary for Section 7.4

1. $\int \frac{1}{2x-1}\,dx$.

Hint: Let $u = 2x - 1$.

Moral. You can integrate anything that looks like $\frac{\text{constant}}{\text{linear}}$!

2. $\int \frac{2x-5}{(x^2-5x)^3}\,dx$.

Hint: Let $u = x^2 - 5x$.

Moral: Always check to see if you can use u-substitution before trying anything fancy!

3. $\int \frac{4x-1}{x^2+5}\,dx$.

Hint: Split up the fraction, then use u-substitution (with $u = x^2 + 5$) on one term and the following **formula** on the other:

$$\int \frac{1}{x^2 + a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C.$$

Moral: You can integrate anything that looks like $\frac{\text{linear}}{\text{x}^2+\text{a}^2}$!

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4. $\int \frac{3x-1}{x^2+6x+11}\,dx$.

Hint: Complete the square in the denominator, *i.e.* $x^2+6x+11 = x^2+6x+9+2 = (x+3)^2+2$. Then let $u = x+3$, and apply the technique in problem 3, above.

Moral: You can integrate anything that looks like $\frac{\text{linear}}{\text{quadratic}}$!

5. $\int \frac{x^3-3x^2+1}{x^2+1}\,dx$.

Hint: Perform **polynomial division**.

Recall: to do long division we get the answer one digit at a time, then multiply, subtract, and get the remainder. Then the answer is (quotient) + $\frac{\text{(remainder)}}{\text{(divisor)}}$.

Example: 1650 divided by 38 is $43\frac{16}{38}$.

To do polynomial division, we do a very similar process with polynomials. Remember to write the terms in descending order by powers, and insert 0 coefficients for missing powers. In other words, the first step should look like

$$x^2+0x+1 \;\Big|\; \overline{x^3-3x^2+0x+1}$$

Then the answer should be a polynomial (the quotient) plus a *proper* rational function (the remainder over x^2+1).

Check to make sure you get $x-3+\frac{-x+4}{x^2+1}$. Integrate.

Moral: When the integrand is an *improper* rational function, perform polynomial division to rewrite the quotient as a polynomial plus a *proper* rational function, then apply the previous techniques.

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