Math 76 Practice Problems for Midterm I - Solutions
§§6.1-7.1

Multiple Choice. *Circle the letter of the best answer.*

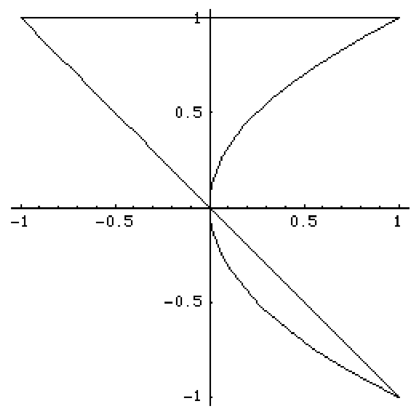
1. What expression best represents the area between $x = y^2$ and $x = -y$ from $y = -1$ to $y = 1$?

(a) $\int_{-1}^{0} (y^2 + y)\, dy + \int_{0}^{1} (-y - y^2)\, dy$

(b) $\boxed{\int_{-1}^{0} (-y - y^2)\, dy + \int_{0}^{1} (y^2 + y)\, dy}$

(c) $\int_{-1}^{1} (y^2 + y)\, dy$

(d) $\int_{-1}^{0} (y^2 - y)\, dy + \int_{0}^{1} (y - y^2)\, dy$



The region described is in two pieces, as shown.

The two curves cross at $y = 0$.

From $y = -1$ to $y = 0$, $x = -y$ is on the right.

From $y = 0$ to $y = 1$, $x = y^2$ is on the right.

Therefore the area is

$$\int_{-1}^{0} (-y - y^2)\, dy + \int_{0}^{1} (y^2 - (-y))\, dy$$
$$= \int_{-1}^{0} (-y - y^2)\, dy + \int_{0}^{1} (y^2 + y)\, dy.$$

2. The volume of the solid formed by rotating the region enclosed by the curves $y = \frac{1}{x^3}$, $y = \frac{1}{x^2}$, and $x = 2$ about the line $x = -1$ is

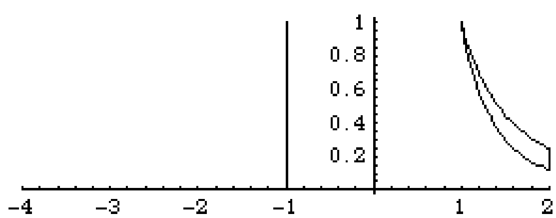
(a) $2\pi \int_{0}^{2} (x + 1) \left(\frac{1}{x^3} - \frac{1}{x^2} \right) dx$

(b) $\boxed{2\pi \int_{1}^{2} (x + 1) \left(\frac{1}{x^2} - \frac{1}{x^3} \right) dx}$

(c) $2\pi \int_{0}^{2} (x - 1) \left(\frac{1}{x^2} - \frac{1}{x^3} \right) dx$

(d) $2\pi \int_{1}^{2} (1 - x) \left(\frac{1}{x^2} - \frac{1}{x^3} \right) dx$

The region being rotated is shown at right with the axis of rotation. It is the same region as in Work and Answer #1. Since the region is formed from functions of x and is being rotated about a vertical axis, we use the **shell method**:



At any x between 1 and 2, the height of the shell is $h = \frac{1}{x^2} - \frac{1}{x^3}$ and the radius is $r = x + 1$. Therefore the volume is

$$2\pi \int_1^2 (x+1)\left(\frac{1}{x^2} - \frac{1}{x^3}\right)\, dx.$$

3. Lois Lane, whose mass is 50 kg, is hanging from a 20-meter rope tied to a crane. Superman is at the top of the crane. In order to rescue Lois, he must pull the rope all the way up to the top of the crane. If the rope has a mass of 10 kg, then the work Superman must do in order to rescue Lois is

(a) 10,780 N

(b) $\boxed{\text{10,780 J}}$

(c) 9,800 N

(d) 9,800 J

The sneaky way to determine the answer is to notice that

- The work done (metric system) is measured in Joules (J), so the answer is either (b) or (d).
- Lois Lane's weight is $50 \cdot 9.8 = 490$ N, so the work required to lift only her is $490 \cdot 20 = 9800$ J, since the rope is 20 m long. So the answer must be (b) since Superman also has to pull the rope up!

But here's how to do the integral:

The rope weighs $10 \cdot 9.8 = 98$ N, or $\frac{98}{20} = 4.9$ Newtons per meter. So if Superman has pulled up x meters of rope, the weight of the rope he has pulled up is $4.9x$. Therefore the weight he is still pulling is $98 - 4.9x = 4.9(20 - x)$ Newtons, in addition to Lois's 490 N. The total work done, then, is

$$\begin{aligned} W &= \int_0^{20} (4.9(20-x) + 490)\ dx \\ &= 4.9 \int_0^{20} (20 - x + 100)\ dx \\ &= 4.9 \int_0^{20} (120 - x)\ dx \\ &= 4.9\left(120x - \frac{1}{2}x^2\right)\Big|_0^{20} \end{aligned}$$

$$\begin{aligned} &= 4.9\left(120 \cdot 20 - \frac{1 \cdot 20^2}{2}\right) - (0 - 0) \\ &= 4.9(2400 - 200) \\ &= 4.9(2200) \\ &= \boxed{10,780 \text{ J}} \end{aligned}$$

4. The temperature (in °F) t hours after 12 noon is $f(t) = 50 + 14\sin(\frac{\pi t}{12})$. The average temperature from 2 pm to 10 pm is

(a) $\frac{1}{8}(500 + \frac{28}{\pi})$ °F

(b) $\boxed{\frac{1}{8}(400 + \frac{14 \cdot 12\sqrt{3}}{\pi})\ °\text{F}}$

(c) $225 + \frac{14}{\pi}$ °F

(d) $450 - \frac{14 \cdot 12}{\pi}$ °F

The average temperature is the average value of the function $f(t)$ from $t = 2$ to $t = 10$, which is

$$\frac{1}{10-2}\int_2^{10} 50 + 14\sin\left(\frac{\pi t}{12}\right)\,dt = \frac{1}{8}\left(50t - \frac{14\cdot 12}{\pi}\cos\left(\frac{\pi t}{12}\right)\right)\Big|_2^{10}$$

$$= \frac{1}{8}\left((50\cdot 10 - \tfrac{14\cdot 12}{\pi}\cos(\tfrac{5\pi}{6})) - (50\cdot 2 - \tfrac{14\cdot 12}{\pi}\cos(\tfrac{\pi}{6}))\right) = \frac{1}{8}\left(400 - \tfrac{14\cdot 12}{\pi}\left(-\tfrac{\sqrt{3}}{2}\right) + \tfrac{14\cdot 12}{\pi}\left(\tfrac{\sqrt{3}}{2}\right)\right)$$

$$= \frac{1}{8}\left(400 + \frac{14\cdot 12\sqrt{3}}{\pi}\right)\ {}^\circ\text{F}.$$

5. A rectangular aquarium 4 ft. wide, 6 ft. long, and 2 ft. high is full of water. If a pump is placed at the top of the tank, the work done in pumping *half* the water out is

(a) 62.5(6) ft.-lb.

(b) $\boxed{\text{62.5(12) ft.-lb.}}$

(c) 62.5(24) ft.-lb.

(d) 62.5(48) ft.-lb.

Using the formula $W = \omega \int_0^b (x+P)A(x)\,dx$ and the weight of water $\omega = 62.5$ lb./ft.^{3}, we get the integral

$$W = 62.5\int_0^1 (x+0)24\,dx,$$

since we are pumping water from a depth of 0 ft. to a depth of 1 ft. (half the water in the tank). $P = 0$ since the pump is at the top of the tank, and $A(x) = 6\cdot 4 = 24$ at all depths x. Evaluating the above integral, we get

$$62.5\int_0^1 24x\,dx = 62.5\cdot 12x^2\Big|_0^1$$

$$= 62.5\cdot 12(1^2 - 0^2) = \boxed{\text{62.5(12) ft.-lb.}}$$

6. $\int_0^1 xe^x\,dx =$

(a) $\boxed{1}$

(b) e

(c) $e - 1$

(d) 0

Using integration by parts, we have

$$u = x \quad\downarrow\quad du = dx \qquad\qquad v = e^x \quad\uparrow\quad dv = e^x\,dx$$

which gives

$$\int_0^1 xe^x\,dx = xe^x\Big|_0^1 - \int_0^1 e^x\,dx$$

$$= xe^x - e^x\Big|_0^1$$

$$= 1e^1 - e^1 - (0e^0 - e^0)$$

$$= e - e - 0 + 1 = \boxed{1}$$

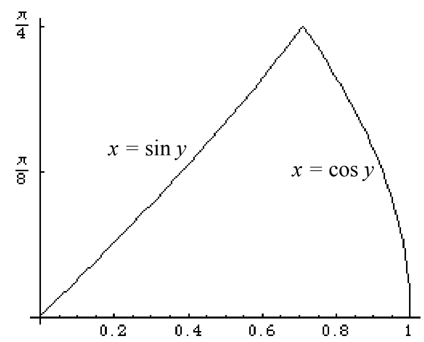
7. The volume of the solid formed by rotating the region shown about the y-axis is

(a) $2\pi \int_0^{\pi/4} y\left(\sin y - \cos y\right)\, dy$

(b) $\pi \int_0^{\pi/4} \left(\cos y - \sin y\right)^2\, dy$

(c) $\boxed{\pi \int_0^{\pi/4} \left(\cos^2 y - \sin^2 y\right)\, dy}$

(d) $2\pi \int_0^{\pi/4} y\left(\cos y - \sin y\right)\, dy$



Since the region is formed by functions of y and is being rotated about a vertical axis, we use the **disk method**:

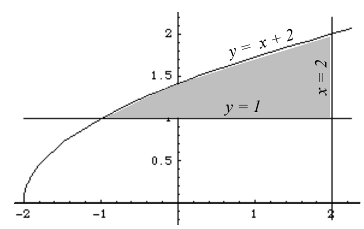
At any y between 0 and $\frac{\pi}{4}$, the outer radius of the disk is $R = \cos y$ and the inner radius of the disk is $r = \sin y$. Therefore the volume is

$$\pi \int_0^{\pi/4} \left((\cos y)^2 - (\sin y)^2\right)\, dy$$
$$= \pi \int_0^{\pi/4} \left(\cos^2 y - \sin^2 y\right)\, dy.$$

Fill-In.

1. If the region enclosed by the curves $y = \sqrt{x+2}$, $y = 1$ and $x = 2$ is rotated about the x-axis, the volume of the resulting solid is $\underline{\frac{9\pi}{2}}$.

The region is shown at right. Since we are rotating a region formed from functions of x about a horizontal axis, it is easiest to use **disks**. The region goes from $x = -1$ to $x = 2$. Also note that $R = \sqrt{x+2}$ and $r = 1$. Thus we get



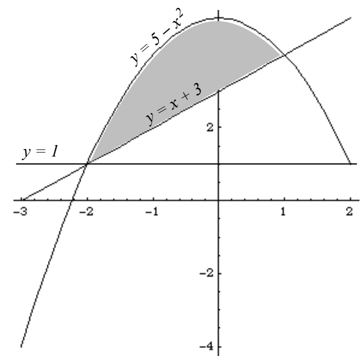
$$V = \pi \int_{-1}^{2} (\sqrt{x+2})^2 - 1^2\, dx$$
$$= \pi \int_{-1}^{2} (x + 2 - 1)\, dx$$
$$= \pi \int_{-1}^{2} (x+1)\, dx$$
$$= \pi \left(\frac{1}{2}x^2 + x\right)\Big|_{-1}^{2}$$
$$= \pi\left(2 + 2 - \left(\frac{1}{2} - 1\right)\right)$$
$$= \pi\left(\frac{9}{2}\right) = \boxed{\frac{9\pi}{2}}$$

This region can also be rewritten in terms of y and the problem solved using shells. See me for help if you want to go over this.

2. If the region enclosed by the curves $y = 5 - x^2$ and $y = x + 3$ is rotated about the line $y = 1$, the volume of the resulting solid is $\underline{\frac{108\pi}{5}}$.

The region is shown at right. Since we are rotating a region formed from functions of x about a horizontal axis, it is easiest to use **disks**. To find where the curves intersect, we set them equal to each other and solve for x:

$$5 - x^2 = x + 3$$
$$x^2 + x - 2 = 0$$
$$(x+2)(x-1) = 0$$
$$x = -2 \quad , \quad x = 1$$



Therefore the region goes from $x = -2$ to $x = 1$. Also note that $R = (5 - x^2) - 1 = 4 - x^2$ and $r = (x+3) - 1 = x + 2$. Thus we get

$$\begin{aligned} V &= \pi \int_{-2}^{1} (4 - x^2)^2 - (x+2)^2 \, dx \\ &= \pi \int_{-2}^{1} (16 - 8x^2 + x^4 - (x^2 + 4x + 4)) \, dx \\ &= \pi \int_{-2}^{1} (12 - 4x - 9x^2 + x^4) \, dx = \pi \left(12x - 2x^2 - 3x^3 + \frac{1}{5}x^5 \right) \Big|_{-2}^{1} \\ &= \pi \left(\left(12 - 2 - 3 + \frac{1}{5} \right) - \left(-24 - 8 + 24 - \frac{32}{5} \right) \right) \\ &= \pi \left(7 + \frac{1}{5} + 8 + \frac{32}{5} \right) = \pi \left(15 + \frac{33}{5} \right) = \boxed{\frac{108\pi}{5}} \end{aligned}$$

3. If 25 N of force are required to keep a spring stretched 20 cm beyond its natural length, then the spring constant for the spring is $k =$ 125 N/m .

   We use Hooke's Law $F(x) = kx$. First we must convert 20 cm to $0.2 = \frac{1}{5}$ m. Then $25 = k \cdot \frac{1}{5}$. Therefore $k = 25 \cdot 5 = \boxed{125 \text{ N/m}}$

4. The average value of the function $f(x) = 4x + 1$ on the interval $[0, 1]$ is 3 .

   We have

$$\begin{aligned} f_{\text{ave}} &= \frac{1}{1-0} \int_0^1 4x + 1 \, dx \\ &= 2x^2 + x \Big|_0^1 \\ &= 2 + 1 - (0 + 0) = \boxed{3} \end{aligned}$$

5. $\displaystyle\int \frac{\ln x}{\sqrt{x}} \, dx =$ $2\sqrt{x} \ln x - 4\sqrt{x} + C$.

   By parts, let

$$u = \ln x \qquad v = 2\sqrt{x}$$

$$du = \frac{1}{x}\,dx \qquad dv = \frac{1}{\sqrt{x}}\,dx$$

Then we have

$$\begin{aligned} \int \frac{\ln x}{\sqrt{x}}\,dx &= 2\sqrt{x}\ln x - 2\int \sqrt{x}\cdot\frac{1}{x}\,dx \\ &= 2\sqrt{x}\ln x - 2\int \frac{1}{\sqrt{x}}\,dx \\ &= \boxed{2\sqrt{x}\ln x - 4\sqrt{x} + C} \end{aligned}$$

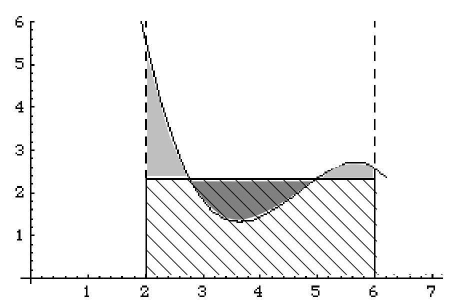
6. To evaluate the integral $\int \cos^{-1}(3x)\,dx$, it is best to use integration by parts with $u = \underline{\cos^{-1}(3x)}$ and $dv = \underline{dx}$.

   This is similar to the problem $\int \sin^{-1}(x)\,dx$ done in class. Please see me if you would like help actually evaluating this integral.

Graph. *More accuracy = more points!*

(a) For the function $f(x)$ graphed at right, sketch a **rectangle** on the same axes whose area is approximately $\displaystyle\int_2^6 f(x)\,dx$.

The top of the rectangle is a horizontal line for which, between $f(x)$ and the line, there is the same amount of area above the line as below.



(b) The average value f_{ave} of $f(x)$ from $x = 2$ to $x = 6$ is approximately $\underline{2.4}$.

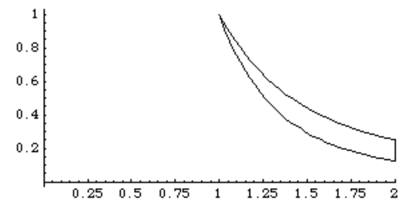
The top of the rectangle is at the average value of $f(x)$, approximately $y = 2.4$.

(c) The approximate value(s) of c so that $f(c) = f_{\text{ave}}$ is/are $\underline{2.8 \text{ and } 5}$ *(list all values)*.

We want the x-values for which $f(x) = f_{\text{ave}} = 2.4$. From the graph, these values are approximately 2.8 and 5.

Work and Answer. *You must show all relevant work to receive full credit.*

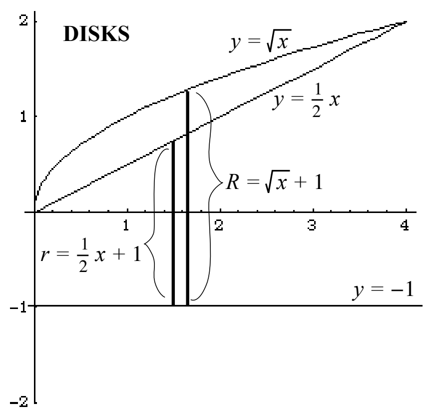
1. Find the area enclosed by the curves $y = \frac{1}{x^2}$, $y = \frac{1}{x^3}$, and $x = 2$.
The region described is shown. It is the same region as in Multiple Choice #2.
Notice that the curve $\frac{1}{x^2}$ is on top between $x = 1$ and $x = 2$. Therefore the area is

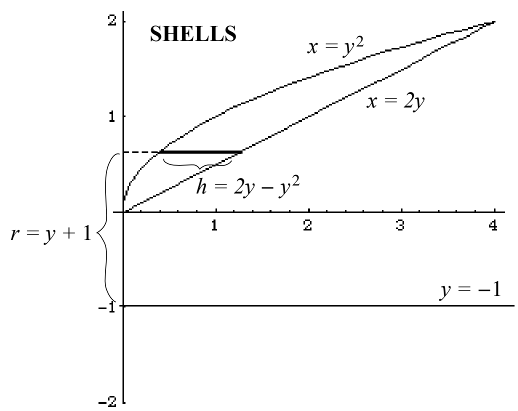


$$\int_1^2 \left(\frac{1}{x^2} - \frac{1}{x^3}\right)\, dx = \int_1^2 (x^{-2} - x^{-3})\, dx$$

$$= -x^{-1} + \frac{1}{2}x^{-2}\Big|_1^2 = -\frac{1}{x} + \frac{1}{2x^2}\Big|_1^2$$

$$= \left(-\frac{1}{2} + \frac{1}{8}\right) - \left(-1 + \frac{1}{2}\right) = \boxed{\frac{1}{8}}$$





2. (a) Use the disk method to find the volume of the solid formed by rotating the region enclosed by the curves $x = y^2$ and $x = 2y$ about the line $y = -1$.

Since we are rotating about a horizontal axis, we will need to rewrite the curves in terms of x. The curves intersect at the points $(0, 0)$ and $(4, 2)$, so we will only need the upper half of the parabola $x = y^2$. Therefore we can rewrite this as $y = \sqrt{x}$ (the positive square root). The region is shown below left, along with R and r. Thus the volume is

$$V = \pi \int_0^4 \left(\sqrt{x} + 1\right)^2 - \left(\frac{1}{2}x + 1\right)^2\, dx$$

$$= \pi \int_0^4 \left(x + 2\sqrt{x} + 1\right) - \left(\frac{1}{4}x^2 + x + 1\right)\, dx$$

$$= \pi \int_0^4 \left(-\frac{1}{4}x^2 + 2\sqrt{x}\right)\, dx = \pi\left(-\frac{1}{12}x^3 + \frac{4}{3}x^{3/2}\right)\Big|_0^4$$

$$= \pi\left(\left(-\frac{1}{12}\cdot 4^3 + \frac{4}{3}\cdot 4^{3/2}\right) - (0 + 0)\right)$$

$$= \pi\left(-\frac{16}{3} + \frac{32}{3}\right) = \boxed{\frac{16\pi}{3}}$$

(b) Use the shell method to find the volume of the solid formed by rotating the region enclosed by the curves $x = y^2$ and $x = 2y$ about the line $y = -1$.

Since we are rotating about a horizontal axis, we can leave the curves in terms of y. The curves intersect at the points $(0,0)$ and $(4,2)$, as before. The region is shown on the previous page (right), along with r and h. Thus the volume is

$$\begin{aligned} V &= 2\pi \int_0^2 (y+1)(2y - y^2)\, dy \\ &= 2\pi \int_0^2 (2y^2 + 2y - y^3 - y^2)\, dy \\ &= 2\pi \int_0^2 (-y^3 + y^2 + 2y)\, dy \\ &= 2\pi \left(-\frac{1}{4}y^4 + \frac{1}{3}y^3 + y^2\right)\Big|_0^2 \\ &= 2\pi \left(\left(-\frac{1}{4}\cdot 2^4 + \frac{1}{3}\cdot 8 + 2^2\right) - (0+0+0)\right) \\ &= 2\pi \left(-4 + \frac{8}{3} + 4\right) = \boxed{\frac{16\pi}{3}} \end{aligned}$$

(c) Should the answers to (a) and (b) be the same? Why or why not?

In (a) and (b) we are rotating the same region about the same axis, so the resulting solids should be the same. Therefore the volumes should be equal.

3. A certain spring has a natural length of 18 in. If 10 lb. of force is needed to keep the spring stretched to a length of 24 in., what is the work done in stretching it to 36 in.?

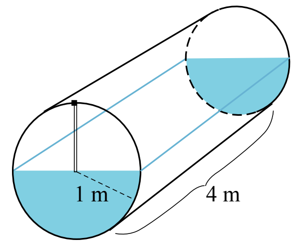
This is a problem where the units are in the English system. However, the distance units are in inches, not feet. So the first thing to do is convert the distances to feet: we have

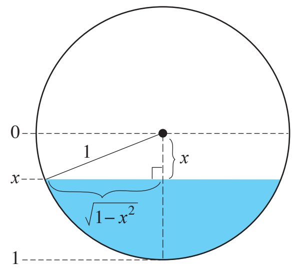
$$\begin{aligned} 18\,\text{in.} &= \tfrac{3}{2}\,\text{ft.} \\ 24\,\text{in.} &= 2\,\text{ft.} \\ 36\,\text{in.} &= 3\,\text{ft.} \end{aligned}$$

Next we use Hooke's Law $F(x) = kx$. We need to use the information in the problem to find k. The problem says that 10 lb. of force are needed to stretch the spring $2 - \frac{3}{2} = \frac{1}{2}$ ft. (remember that x in Hooke's Law is the number of feet ***beyond** the natural length*). So $10 = k \cdot \frac{1}{2}$. Therefore $k = 20$. So the work done to stretch it to 3 ft. (= 1.5 ft. beyond the natural length) is

$$\begin{aligned} W &= \int_0^{1.5} 20x\, dx \\ &= 10x^2 \Big|_0^{1.5} \\ &= 10\cdot(1.5)^2 - 0 = 10\cdot 2.25 = \boxed{22.5\,\text{ft.-lb.}} \end{aligned}$$

4. A tank in the shape of a cylinder on its side is half full of water. The pump is at the top of the tank, as shown below. Set up, **but do not evaluate,** an integral for the work done in pumping all the water out of the tank.





We set $x = 0$ to be the initial water level, as shown above. The pump is 1 m above that, so $P = 1$ (see the formula $W = \omega \int_0^b (x+P)A(x)\,dx$ on the formula list). Since we are in the metric system, $\omega = 9800$. Finally, the surface area $A(x)$ of the water at each depth x is a rectangle 4 m long and $2\sqrt{1-x^2}$ m wide (using the Pythagorean Theorem; see the picture), so $A(x) = 8\sqrt{1-x^2}$ square meters. Therefore the work done is

$$\begin{aligned} W &= 9800 \int_0^1 (x+1) \cdot 8\sqrt{1-x^2}\,dx \\ &= \boxed{78,400 \int_0^1 (x+1)\sqrt{1-x^2}\,dx} \end{aligned}$$

($9800 \cdot 8 = 78,400$).
In case you are interested in evaluating the above integral (it would be great practice!), here's the solution:

First distribute the $x+1$ to get

$$78,400 \left[\int_0^1 x\sqrt{1-x^2}\,dx + \int_0^1 \sqrt{1-x^2}\,dx \right].$$

This is similar to §6.3 #42 from the homework. For the first integral we use a u-substitution: let $u = 1 - x^2$. Then $du = -2x\,dx$. We also need to change the x-limits to u-limits. When $x = 0$, $u = 1 - 0^2 = 1$, and when $x = 1$, $u = 1 - 1^2 = 0$. Therefore we get

$$\begin{aligned} \int_0^1 x\sqrt{1-x^2}\,dx &= -\frac{1}{2}\int_1^0 \sqrt{u}\,du && \text{(futzing the } -2\text{)} \\ &= \frac{1}{2}\int_0^1 \sqrt{u}\,du && \text{(switching the limits and getting rid of the negative)} \\ &= \frac{1}{2} \cdot \frac{2}{3} u^{3/2} \Big|_0^1 \\ &= \frac{1}{3}(1-0) = \frac{1}{3}. \end{aligned}$$

For the second integral we can use geometry. $\int_0^1 \sqrt{1-x^2}\,dx$ represents the area of $\frac{1}{4}$ of a circle of radius 1, so

$$\int_0^1 \sqrt{1-x^2}\,dx = \frac{1}{4}\cdot\pi\cdot 1^2 = \frac{\pi}{4}.$$

Therefore the final answer for the work done would be

$$W = 78,400\left(\frac{1}{3}+\frac{\pi}{4}\right).$$

5. Find the average value of the function $f(x) = x^3 - 2x + 1$ on the interval $[-2, 1]$.

   We have

$$\begin{aligned} f_{\text{ave}} &= \frac{1}{1-(-2)}\int_{-2}^{1}(x^3-2x+1)\,dx \\ &= \frac{1}{3}\left(\frac{1}{4}x^4 - x^2 + x\right)\Big|_{-2}^{1} \\ &= \frac{1}{3}\left(\left(\frac{1}{4}-1+1\right)-\left(\frac{1}{4}\cdot 16 - 4 - 2\right)\right) \\ &= \frac{1}{12}+\frac{1}{3}\cdot 2 \\ &= \frac{1}{12}+\frac{8}{12} = \frac{9}{12} = \boxed{\frac{3}{4}} \end{aligned}$$

6. Evaluate the integral $\int x\sin 3x\,dx$.

   Using integration by parts, we have

$$\begin{array}{cc} u = x & v = -\frac{1}{3}\cos 3x \\ \downarrow & \uparrow \\ du = dx & dv = \sin 3x\,dx \end{array}$$

   which gives

$$\begin{aligned} \int x\sin 3x\,dx &= -\frac{1}{3}x\cos 3x + \frac{1}{3}\int \cos 3x\,dx \\ &= \boxed{-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x + C} \end{aligned}$$

7. Evaluate the integral $\int x\sin^{-1}(x^2)\,dx$.

   This is similar to a problem we did in homework, but first we must make the u-substitution (actually we will use t instead of u because we will soon be using integration by parts, and we don't want to confuse the u's): let $t = x^2$; then $dt = 2x\,dx$, and we get $\frac{1}{2}\int \sin^{-1}(t)\,dt$. Now using integration by parts, we have

$$\begin{array}{cc} u = \sin^{-1}(t) & v = t \\ \downarrow & \uparrow \\ du = \frac{1}{\sqrt{1-t^2}}\,dt & dv = dt \end{array}$$

which gives

$$\frac{1}{2}\int \sin^{-1}(t)\,dt = \frac{1}{2}\left(t\sin^{-1}(t) - \int \frac{t}{\sqrt{1-t^2}}\right)\,dt$$

(Now we use one final substitution for the remaining integral: $u = 1 - t^2$. Then $du = -2t\,dt$.)

$$\begin{aligned}
&= \frac{1}{2}\left(t\sin^{-1}(t) + \frac{1}{2}\int \frac{1}{\sqrt{u}}\,du\right) \\
&= \frac{1}{2}\left(t\sin^{-1}(t) + \frac{1}{2}\cdot 2u^{1/2}\right) + C \\
&= \frac{1}{2}\left(t\sin^{-1}(t) + \sqrt{1-t^2}\right) + C. \qquad \text{Finally we go back to } x\text{'s:} \\
&= \boxed{\frac{1}{2}x^2\sin^{-1}(x^2) + \frac{1}{2}\sqrt{1-x^4} + C}
\end{aligned}$$

8. Evaluate the integral $\int e^x \sin x\,dx$.

   Using integration by parts, we have

$$\begin{array}{ccc}
u = e^x & \qquad & v = -\cos x \\
\downarrow & & \uparrow \\
du = e^x\,dx & & dv = \sin x\,dx
\end{array}$$

   which gives

$$\int e^x \sin x\,dx = -e^x\cos x + \int e^x \cos x\,dx.$$

   This is similar to a problem that we did in class. Remember that we had to do parts twice and solve for the integral. So here's the second application of integration by parts:

$$\begin{array}{ccc}
u = e^x & \qquad & v = \sin x \\
\downarrow & & \uparrow \\
du = e^x\,dx & & dv = \cos x\,dx
\end{array}$$

   We get $-e^x\cos x + \left(e^x \sin x - \int e^x \sin x\right)$. Now we are ready to solve for the integral; we have

$$\int e^x \sin x\,dx = -e^x\cos x + e^x\sin x - \int e^x \sin x,$$

   so

$$2\int e^x \sin x\,dx = -e^x\cos x + e^x\sin x + C.$$

   Therefore

$$\int e^x \sin x\,dx = \boxed{-\frac{1}{2}e^x\cos x + \frac{1}{2}e^x\sin x + C}$$

9. Evaluate the integral $\int x^2 \ln(x^3)\,dx$.

Again, this is similar to problems we have done before. First we substitute $u = x^3$. Then $du = 3x^2\,dx$, and we have $\frac{1}{3}\int \ln u\,du$. We showed in class that $\int \ln t\,du = t\ln t - t + C$ (you can rederive it by letting $u = \ln t$ and $dv = dt$ and using parts); therefore we get

$$\begin{aligned}\frac{1}{3}\int \ln u\,du &= \frac{1}{3}(u\ln u - u) + C \\ &= \boxed{\frac{1}{3}(x^3\ln(x^3) - x^3) + C}\end{aligned}$$

If you are feeling clever, convince yourself that the above simplifies to $x^3\left(\ln x - \frac{1}{3}\right) + C$!