Math 76 Practice Problems for Midterm III
Chapter 11

DISCLAIMER. This collection of practice problems is *not* guaranteed to be identical, in length or content, to the actual exam. You may expect to see problems on the test that are not exactly like problems you have seen before.

On the actual exam you will have more room to work the problems. You will see directions similar to these:

1. Please read directions carefully. Raise your hand if you are not sure what a problem is asking.

2. *You must explain your work thoroughly and unambiguously to receive full credit on questions or parts of questions designated as* **Work and Answer**.

3. **No calculators or notes are allowed on this exam.**

4. You have 65 minutes to complete your test, unless announced otherwise. Do not spend too long on any one problem. You do not have to do the problems in order. Do the easy ones first. Do not attempt the bonus question until you have completed the rest of the test. Before turning in your test, please make sure you have answered and double-checked all the questions.

5. If you need scratch paper, please raise your hand. You may not use your own paper. When you have finished your exam, please turn in any scratch paper you use.

6. Write your solutions in the space provided for each problem, or provide specific instructions as to where your work is to be found. *Make it clear what you want and don't want graded.* Your final answers should be boxed or circled.

7. Unless directed otherwise, only EXACT ANSWERS will receive full credit (i.e. $\sqrt{2}$, not 1.414).

8. In word problems, give units on all answers (e.g. feet, grams, gallons).

9. Don't stress! I'm rooting for you!

Multiple Choice. *Circle the letter of the best answer.*

1. The nth term of the sequence $\{-3, 4, 11, 18, 25, \ldots\}$, counting $a_1 = -3$ as the first term, is

   (a) $a_n = 5n - 2$ (c) $a_n = n^2 - 4$

   (b) $a_n = 7n - 10$ (d) $a_n = -3n + 7$

2. $\displaystyle\sum_{n=1}^{\infty} 3\left(\tfrac{1}{2}\right)^n =$

   (a) 6   (c) $\dfrac{3}{2}$

   (b) 3   (d) ∞ (diverges)

3. The series $\displaystyle\sum_{n=1}^{\infty} \frac{2}{3^{n+2}}$

    (a) converges to $\dfrac{8}{9}$ (c) converges to 3

    (b) converges to $\dfrac{1}{9}$ (d) converges to 9

4. $\displaystyle\sum_{n=3}^{\infty} \left(\frac{2}{n} - \frac{2}{n+1} \right) =$

    (a) 0   (c) $\dfrac{2}{3}$

    (b) $\dfrac{1}{6}$ (d) ∞ (diverges)

5. To determine whether or not the series $\displaystyle\sum_{n=2}^{\infty} \frac{5n^3}{1 - 2n + n^4}$ converges, the limit comparison test may be used with comparison series $\displaystyle\sum b_n =$

    (a) $\displaystyle\sum \frac{1}{n}$ (c) $\displaystyle\sum \frac{5}{n^4}$

    (b) $\displaystyle\sum 5n^3$ (d) none; the limit comparison test cannot be used

6. The series $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\sqrt{n}}{n^2 - 4\sqrt{n} - 1}$

    (a) converges absolutely (AC)

    (b) converges conditionally (CC)

    (c) diverges

7. The series $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\sqrt{n}}{n - 4\sqrt{n} - 1}$

    (a) converges absolutely (AC)

    (b) converges conditionally (CC)

    (c) diverges

8. The series $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{10^n}{7\,n!}$

   (a) converges absolutely (AC)

   (b) converges conditionally (CC)

   (c) diverges

9. The series $\displaystyle\sum_{n=2}^{\infty}\left(\frac{2n^2+1}{n^2+5n-6}\right)^n$

   (a) converges absolutely (AC)

   (b) converges conditionally (CC)

   (c) diverges

10. The interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}(x-1)^n$ is

   (a) $[0,1]$ (c) $(0,2]$

   (b) $(0,1)$ (d) $[0,2)$

11. A power series representation for the function $f(x)=\dfrac{3}{4-x}$ is

   (a) $\displaystyle\sum_{n=0}^{\infty}\frac{3}{4}x^n$ (c) $\displaystyle\sum_{n=0}^{\infty}\frac{3}{4^{n+1}}x^n$

   (b) $\displaystyle\sum_{n=0}^{\infty}\left(\frac{3}{4}\right)^{n+1}x^n$ (d) $\displaystyle\sum_{n=0}^{\infty}3(4-x)^n$

12. The Maclaurin series for the function $f(x)=x^3\cos(4x^2)$ is

   (a) $\displaystyle\sum_{n=0}^{\infty}\frac{(-16)^n}{(2n)!}x^{4n+3}$ (c) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}(4x^2)^n$

   (b) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}x^{2n+3}$ (d) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}x^{4n^2+3}$

13. The Maclaurin series for the function $f(x)=\sqrt{3-x}$ is

   (a) $\displaystyle\sum_{n=0}^{\infty}\binom{-\frac{1}{2}}{n}\frac{1}{3^n}x^n$ (c) $\displaystyle\sum_{n=0}^{\infty}\binom{\frac{1}{3}}{n}x^n$

   (b) $\displaystyle\sum_{n=0}^{\infty}\binom{\frac{1}{2}}{n}\sqrt{3}\left(-\frac{1}{3}\right)^n x^n$ (d) $\displaystyle\sum_{n=0}^{\infty}\binom{n}{\frac{1}{2}}\frac{(-1)^n}{3^n}x^n$

3

Fill-In.

1. *Circle the best answer. On the line, indicate one valid test that can be applied to get your answer. You may choose from the following list:*

- divergence test
- p-series test
- geometric series test

- integral test
- direct comparison test
- limit comparison test

- alternating series test
- ratio test
- root test

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{4}{n^3}$ (**converges | diverges**)   Test: _____

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^3 - 1}{3n^2}$ (**converges | diverges**)   Test: _____

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{3\sqrt{n}}{n^2 - 3n + 1}$ (**converges | diverges**)   Test: _____

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\tan^{-1}(n)}$ (**converges | diverges**)   Test: _____

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{10^n}{(5n)!}$ (**converges | diverges**)   Test: _____

2. $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{n^2 + 1}(x+3)^{n-1}$ is a power series centered at _____ .

3. The radius of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} x^n$ is _____ .

4. A power series representation of $6\tan^{-1}(x^2)$ is _____ .

5. The Maclaurin series of $\cos(3x)$ is _____ .

6. The Taylor series of $\cos(3x)$ centered at π is _____ .

7. $\displaystyle\binom{7}{3} =$ _____ .

8. $\displaystyle\binom{\frac{4}{3}}{5} =$ _____ .

4

Work and Answer. *You must show all relevant work to receive full credit.*

1. For the sequence $\{2, -\frac{4}{3}, \frac{8}{9}, -\frac{16}{27}, \ldots\}$,

   (a) Find a formula for the n-th term a_n of the sequence, assuming $a_0 = 2$.

   (b) *Circle the best answer.* The sequence $\{a_n\}_{n=0}^{\infty}$ above (**converges** | **diverges**).

2. Find the sum of the series $\displaystyle\sum_{n=-1}^{\infty} \frac{2 \cdot 3^n}{4^{n-1}}$.

3. Find the sum of the series $\displaystyle\sum_{n=2}^{\infty} \frac{n+1}{n^3 - n}$.

4. Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{3^n \sin n}{n!}$ is absolutely convergent (AC), conditionally convergent (CC), or divergent.

5. (a) Find a power series representation for the function $f(x) = \ln(2 + 3x)$.

   (b) Find the interval of convergence.

6. (a) Write the Taylor series for the function $f(x) = \sqrt{x}$ centered at 1.

   (b) Find the radius of convergence.

   (c) Estimate $\sqrt{1.4}$ using the first three terms of the Taylor series.

7. Estimate $\displaystyle\int_0^1 e^{x^2}\, dx$ using the first two terms of the Maclaurin series expansion.

8. Estimate $\displaystyle\int_0^1 \sin x^2\, dx$ using the first two terms of the Maclaurin series expansion.

9. (a) Write out and simplify $\dbinom{\frac{4}{5}}{n}$ for $n \geq 2$.

   (b) Write the binomial series of $(1 - 2x^3)^{4/5}$ and use (a) to simplify.

Some kind of **BONUS.**