Math 75A Practice Final

DISCLAIMER. This collection of practice problems is *not* guaranteed to be identical, in length or content, to the actual exam. You may expect to see problems on the test that are not exactly like problems you have seen before.

On the actual exam you will see directions similar to these:

1. Please read directions carefully. Raise your hand if you are not sure what a problem is asking.
2. *You must explain your work thoroughly and unambiguously to receive full credit on questions or parts of questions designated as **Work and Answer**.*
3. **No calculators or notes are allowed on this exam.**
4. You have 2 hours to complete your test, unless announced otherwise. Do not spend too long on any one problem. You do not have to do the problems in order. Do the easy ones first. Do not attempt the bonus question until you have completed the rest of the test. Before turning in your test, please make sure you have answered and double-checked all the questions.
5. If you need scratch paper, please raise your hand. You may not use your own paper. When you have finished your exam, please turn in any scratch paper you use.
6. For **Work and Answer** problems, write your solutions in the space provided for each problem, or provide specific instructions as to where your work is to be found. *Make it clear what you want and don't want graded.* Your final answers should be boxed or circled.
7. Don't stress! I'm rooting for you!

Multiple Choice. (55 points) *Circle the letter of the best answer.*

1. The vertical asymptotes for the graph of the function $f(x) = \dfrac{x^2+4x+4}{x^2+x-2}$ are

(a) $x = -2$ and $x = 1$

(b) $x = -2$ only

(c) $x = -1$ and $x = 2$

(d) $x = 1$ only

(e) $y = 1$ only

2. If $e^{2t} = 5$, then $t =$

(a) $\dfrac{e^2}{5}$

(b) $\dfrac{\ln 5}{2}$

(c) $\ln \dfrac{5}{2}$

(d) $\dfrac{5}{e^2}$

(e) $\dfrac{2}{\ln 5}$

3. Evaluate $\lim_{x \to 0} \dfrac{\sqrt{4+x}-2}{x}$.

(a) 0

(b) $\dfrac{1}{4}$

(c) $\dfrac{1}{2}$

(d) 1

(e) does not exist

4. Let $g(t) = t^3 \cos t$. Find $g'(t)$.

(a) $3t^2 \cos t - t^3 \sin t$

(b) $-3t^2 \sin t$

(c) $3t^2 \sin t$

(d) $-3t^2 \sin t + t^3 \cos t$

(e) $3t^2 \cos t + t^3 \sin t$

5. At $x = 3$ the graph of the function $f(x) = \dfrac{2x-6}{x-3}$

(a) has a hole

(b) has a vertical asymptote

(c) is at the point $(3, 2)$

(d) has a corner

(e) has a vertical tangent line

6. The inverse function of $f(x) = 5x^3 - 4$ is $f^{-1}(x) =$

(a) $\dfrac{\sqrt[3]{x}+4}{5}$

(b) $\dfrac{5}{\sqrt[3]{x-4}}$

(c) $\sqrt[3]{\dfrac{x+4}{5}}$

(d) $\dfrac{\sqrt[3]{x+4}}{5}$

(e) $f(x)$ does not have an inverse function

7. If $f(3) = 4$, $g(3) = 2$, $f'(3) = -6$, and $g'(3) = 5$, find the derivative of the quotient function $\left(\dfrac{f}{g}\right)(x)$ at $x = 3$, that is, $\left(\dfrac{f}{g}\right)'(3)$.

(a) -8

(b) -3

(c) $-\dfrac{6}{5}$

(d) 2

(e) 8

8. Evaluate $\lim_{x \to 2} \dfrac{x^2-4}{x^2+x-6}$.

(a) 0

(b) 1

(c) $\frac{2}{3}$

(d) $\frac{4}{5}$

(e) does not exist

9. If $f(x) = \sin(x^2)$, then $f'(x) =$

(a) $\cos(2x)$

(b) $-2x\cos(x^2)$

(c) $\sin(2x) + \cos(x^2)$

(d) $\cos(x^2)$

(e) $2x\cos(x^2)$

10. The horizontal asymptote(s) of the function $f(x) = \dfrac{-7x^3 + 5x - 1}{\sqrt{9x^6 + 2}}$ is/are

(a) $y = 0$ only

(b) $y = \frac{7}{3}$ only

(c) $y = \frac{7}{3}$ and $y = -\frac{7}{3}$

(d) $y = -\frac{7}{3}$ only

(e) $f(x)$ has no horizontal asymptotes

11. If $f(x) = \sin(x)$, then $f'(x) =$

(a) $\dfrac{\sin(x+h) - \sin(x)}{h}$

(b) $\dfrac{\sin(x) + h - \sin(x)}{h}$

(c) $\lim_{h\to 0} \dfrac{\cos(x+h) - \cos(x)}{h}$

(d) $\lim_{x\to 0} \dfrac{\sin(x+h) - \sin(x)}{h}$

(e) $\lim_{h\to 0} \dfrac{\sin(x+h) - \sin(x)}{h}$

Fill-In. (35 points)

1. The domain of the function $f(x) = \sqrt{3x - 2}$ is __________________________

2. The range of the function $g(t) = 5\cos t$ is __________________________

3. $\lim_{x\to 4^-} \dfrac{2}{x-4} =$ _____________

4. The table below gives the distance traveled on a straight stretch of highway t minutes after 2:30pm. For example, at 2:35pm the distance traveled was 4.2 miles. The average speed of the vehicle *(with units – e.g. grams, gallons, etc.)* from 2:32 to 2:37 is

________________________________.

t	0	1	2	3	4	5	6	7
miles	0	0.9	1.7	2.8	3.8	4.2	5.0	5.9

5. If $\sin\theta = -\dfrac{2}{3}$ and θ is in quadrant IV, then

   (a) $\cos\theta =$ ____________

   (b) $\tan\theta =$ ____________

   (c) $\sec\theta =$ ____________

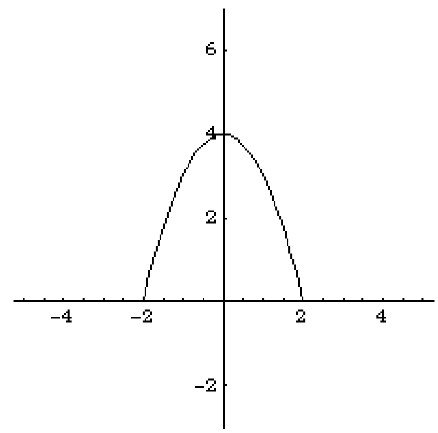
   (d) $\csc\theta =$ ____________

   (e) $\cot\theta =$ ____________

Graphs. (20 points)

1. The graph of $f(x)$ is shown. On the same axes, sketch a graph of $f(x-3)$.

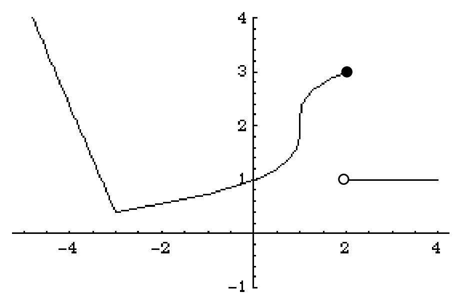
   More accuracy = more points!



2. The graph of the function $f(x)$ shown at right is

   (a) discontinuous at $x =$ ______________________ (list all x-values)

   (b) not differentiable (does not have a derivative) at $x =$ ______________________ (list all x-values)



Work and Answer. (90 points) *You must show all relevant work to receive full credit.*

1. (15 points) If $f(x) = \sqrt{x^2+1}$ and $g(x) = \dfrac{1}{x^2-1}$, find $(f \circ g)(x)$.

2. (15 points) Find the derivative of the function $f(t) = 10t^{18} - \cos t + \tan\left(\frac{1}{t}\right)$.

3. (15 points) Find the slope of the tangent line to the curve $f(x) = 6xe^{5x^2}$ at the point $(2, 1)$.

4. (15 points) A coin is tossed off the top of a building at the new Mars space station (sometime in the future). Its height in meters after t seconds is $h(t) = 20 - 10t + 1.8t^2$. Find the velocity of the penny after 2 seconds. *You may assume that the penny is still in the air after 2 seconds. Be sure to give units (e.g. feet, kilograms, etc.).*

5. (15 points) Suppose

$$\begin{aligned} f(1) &= 5 & \qquad g(1) &= 2 \\ f'(1) &= 3 & g'(1) &= 7 \\ f'(2) &= -10 & g'(5) &= -4 \end{aligned}$$

   (a) Find $(f \circ g)'(1)$.
   (b) Find $(g \circ f)'(1)$.

6. (15 points) 1000 bacteria are placed in a jar with enough food so that the population triples every 10 days.
   (a) Write a model for the population t days after the bacteria are placed in the jar.
   (b) Suppose the jar is big enough so that in 60 days it will be full. When will the jar be 1/3 full?

Some kind of **BONUS.** (5 points)