Math 75B Practice Problems for Midterm II
Ch. 16, 17, 12 (E), §§4.1-4.5, 2.8 (S)

DISCLAIMER. This collection of practice problems is *not* guaranteed to be identical, in length or content, to the actual exam. You may expect to see problems on the test that are not exactly like problems you have seen before.

On the actual exam you will have more room to work the problems. You will see directions similar to these:

1. Please read directions carefully. Raise your hand if you are not sure what a problem is asking.
2. *You must explain your work thoroughly and unambiguously to receive full credit on questions or parts of questions designated as* ***Work and Answer****.*
3. **No calculators or notes are allowed on this exam.**
4. You have 50 minutes to complete your test, unless announced otherwise. Do not spend too long on any one problem. You do not have to do the problems in order. Do the easy ones first. Do not attempt the bonus question until you have completed the rest of the test. Before turning in your test, please make sure you have answered and double-checked all the questions.
5. If you need scratch paper, please raise your hand. You may not use your own paper. When you have finished your exam, please turn in any scratch paper you use.
6. Write your solutions in the space provided for each problem, or provide specific instructions as to where your work is to be found. *Make it clear what you want and don't want graded.* Your final answers should be boxed or circled.
7. Unless directed otherwise, only EXACT ANSWERS will receive full credit (i.e. $\sqrt{2}$, not 1.414).
8. In word problems, give units on all answers (e.g. feet, grams, gallons).
9. Don't stress! I'm rooting for you!

True or False. Circle **T** if the statement is *always* true; otherwise circle **F**.

1. The absolute maximum value of $f(x) = \frac{1}{x}$ on the interval $[2, 4]$ is 2. **T** **F**

2. If $f(x)$ is a continuous function and $f(3) = 2$ and $f(5) = -1$, then $f(x)$ has a root between 3 and 5. **T** **F**

3. The function $g(x) = 2x^3 - 12x + 5$ has 5 real roots. **T** **F**

4. If $h(x)$ is a continuous function and $h(1) = 4$ and $h(2) = 5$, then $h(x)$ has no roots between 1 and 2. **T** **F**

5. The only x-intercept of $f(x) = x^3 - x^2 + 2x - 2$ is $(1, 0)$. *(Challenge problem!)* **T** **F**

Multiple Choice. *Circle the letter of the best answer.*

1. The function $f(x) = x^4 - 6x^2$ is increasing on the intervals

   (a) $(0, \sqrt{3})$ only

   (b) $(-\infty, -\sqrt{3})$ and $(0, \sqrt{3})$ only

   (c) $(\sqrt{3}, \infty)$ only

   (d) $(-\sqrt{3}, 0)$ and $(\sqrt{3}, \infty)$ only

2. The function $f(x) = \cos x - x$

   (a) is an even function

   (b) is an odd function

   (c) is neither an even nor an odd function

3. The function $f(x) = x^4 - 6x^2$ is concave down on the intervals

   (a) $(-1, 1)$ only

   (b) $(-\sqrt{3}, \sqrt{3})$ only

   (c) $(-\infty, -1)$ and $(1, \infty)$ only

   (d) $(1, \sqrt{3})$ only

4. The linear approximation of $f(x) = \sqrt{5 - x}$ at $x = 1$ is

   (a) $y = -\frac{1}{4}x + \frac{9}{4}$

   (b) $y = -\frac{3}{4}x + \frac{7}{4}$

   (c) $y = \frac{1}{4}x + \frac{7}{4}$

   (d) $y = -\frac{3}{4}x + \frac{9}{4}$

5. Suppose $g(x)$ is a polynomial function such that $g(-1) = 4$ and $g(2) = 7$. Then there is a number c between -1 and 2 such that

   (a) $g(c) = 1$

   (b) $g'(c) = 1$

   (c) $g(c) = 0$

   (d) $g'(c) = 0$

Fill-In.

1. If $f(x)$ is continuous and differentiable on the interval $[2, 4]$ and $f(2) = -1$ and $f(4) = 3$, then there is a number c between 2 and 4 such that *(check **all** that apply)*

   ______ $f(c) = 0$ ______ $f(c) = 2$ ______ $f(c) = 4$

   ______ $f'(c) = 0$ ______ $f'(c) = 2$ ______ $f'(c) = 4$

2. If a polynomial function $f(x)$ has 3 solutions to the equation $f'(x) = 0$, then $f(x)$ has at most ________ real roots.

3. A contractor has 80 ft. of fencing with which to build three sides of a rectangular enclosure. In order to enclose the largest possible area, the dimensions of the enclosure should be _______ $\times$ _______.

4. Using a tangent line approximation, $\sqrt[3]{126.5} \approx$ _____________.

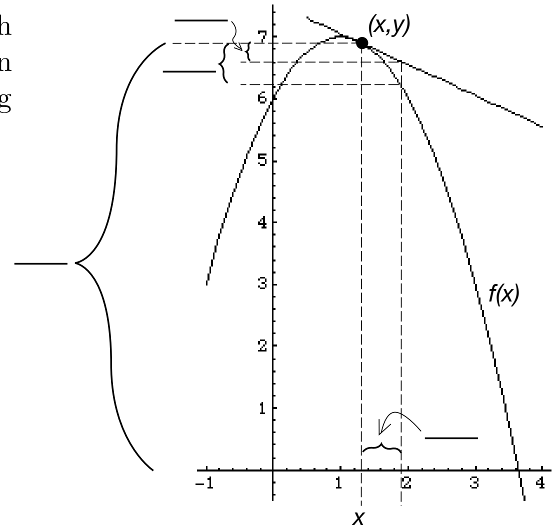
Graphs.

1. For the function $h(x) = \frac{x+5}{x^2-9}$,
   (a) Find the equations of any vertical and/or horizontal asymptotes
   (b) Find the y-intercept
   (c) Find the x-intercept
   (d) Find the critical **points** and intervals of increase/decrease
   (e) On the (invisible) axes at right, sketch an accurate graph of $h(x)$.

2. For the function $g(x) = \frac{2}{3}x^3 - 2x^2$,
   (a) Find the critical **points** and intervals of increase/decrease
   (b) Find the inflection **points** and intervals of concave up/concave down
   (c) Discuss any symmetry $g(x)$ may or may not have
   (d) Find the equations of any vertical and/or horizontal asymptotes
   (e) Find the y-intercept
   (f) On the (invisible) axes at right, sketch an accurate graph of $g(x)$.

3. On the (invisible) axes at right, sketch a graph of a function $f(x)$ satisfying all of the following properties:
   - $f(x)$ is an even function
   - $(3, 0)$ is a critical point of $f(x)$
   - $(4, -1)$ is an inflection point of $f(x)$
   - $f(x)$ has a vertical asymptote at $x = 0$ and a horizontal asymptote at $y = -2$
   - $f(x)$ is increasing on the intervals $(-\infty, -3)$ and $(0, 3)$
   - $f(x)$ is concave up on the intervals $(-\infty, -4)$ and $(4, \infty)$

4. The graph of a function $f(x)$ is shown, along with the linear approximation at a point (x, y). Fill in each blank with the correct label from the following list: dx, y, dy, and Δy.



Work and Answer. *You must show all relevant work to receive full credit.*

1. Find the absolute maximum and the absolute minimum values of the function $f(x) = \frac{\ln x}{x}$ on the interval $[1, e^2]$.

2. If 1200 cm^2 of sheet metal is available to make a box with a square base and open top, find the largest possible volume of the box.

3. A cone-shaped roof with base radius $r = 6$ ft. is to be covered with a 0.5-inch layer of tar. Use differentials to estimate the amount of tar required (you may use the formula $V(r) = \frac{2}{9}\pi r^3$ for the volume of the piece of the house covered by the roof).

4. Prove that the function $f(x) = -x^3 - 6x + 1$ has exactly one real root by completing the following:

   (a) Use the Intermediate Value Theorem to show that the function $f(x) = -x^3 - 6x + 1$ has *at least* one real root.

   (b) Use Rolle's Theorem to show that the function $f(x) = -x^3 - 6x + 1$ has *at most* one real root.

Some kind of **BONUS.**