Math 75B Practice Midterm I
Ch. 9-11, 14-C.2 (Ebersole), §§2.6, 2.7, 3.3, 3.5, 3.7 (Stewart)

DISCLAIMER. This collection of practice problems is *not* guaranteed to be identical, in length or content, to the actual exam. You may expect to see problems on the test that are not exactly like problems you have seen before.

On the actual exam you will see directions similar to these:

1. Please read directions carefully. Raise your hand if you are not sure what a problem is asking.
2. *You must explain your work thoroughly and unambiguously to receive full credit on questions or parts of questions designated as **Work and Answer**.*
3. **No calculators or notes are allowed on this exam.**
4. You have 50 minutes to complete your test, unless announced otherwise. Do not spend too long on any one problem. You do not have to do the problems in order. Do the easy ones first. Do not attempt the bonus question until you have completed the rest of the test. Before turning in your test, please make sure you have answered and double-checked all the questions.
5. If you need scratch paper, please raise your hand. You may not use your own paper. When you have finished your exam, please turn in any scratch paper you use.
6. Write your solutions in the space provided for each problem, or provide specific instructions as to where your work is to be found. *Make it clear what you want and don't want graded.* Your final answers should be boxed or circled.
7. Don't stress! I'm rooting for you!

True or False. Circle **T** if the statement is *always* true; otherwise circle **F**.

1. If $g(x) = x^x$, then $g'(x) = x \cdot x^{x-1}$. **T** **F**
2. $\dfrac{d}{dt}\cos^{-1}(3t^2+1) = -\dfrac{6t}{\sqrt{1-(3t^2+1)^2}}$. **T** **F**
3. $\dfrac{d}{dt}\tan^{-1}(3t^2+1) = \dfrac{6t}{\sqrt{1-(3t^2+1)^2}}$. **T** **F**
4. The range of the function $f(x) = \sin^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. **T** **F**
5. The range of the function $f(x) = \cos^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. **T** **F**
6. $\dfrac{d}{dt}(\log(3t^2+1)) = \dfrac{(6\ln 10)t}{3t^2+1}$. **T** **F**
7. If $3x^2y = \tan(y^2)$, then $\dfrac{dy}{dx} = \dfrac{-6xy}{3x^2 - 2y\sec^2(y^2)}$. **T** **F**
8. The limit $\displaystyle\lim_{x\to 0}\left(\frac{1}{x}\right)^{\cos x}$ is an indeterminate form. **T** **F**

Multiple Choice. *Circle the letter of the best answer.*

1. If $x^2 - y^2 = 4$, then $\dfrac{dy}{dx} =$

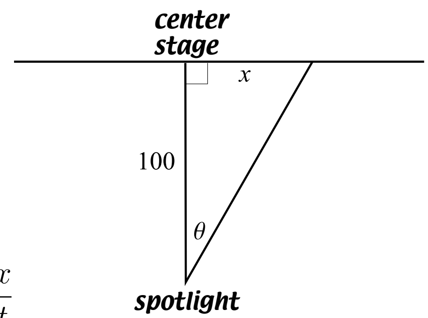
(a) $\dfrac{y}{x}$

(b) $-\dfrac{y}{x}$

(c) $\dfrac{x}{y}$

(d) $-\dfrac{x}{y}$

2. A ballet dancer tiptoes across a stage while a spotlight 100 ft. away from center stage follows her, as shown. An equation to describe the relationship between the rate of change of the angle θ and the dancer's speed is



(a) $\sec\theta \cdot \dfrac{d\theta}{dt} = \dfrac{100}{x}\dfrac{dx}{dt}$

(b) $\tan\theta = \dfrac{x}{100}$

(c) $\cos\theta \cdot \dfrac{d\theta}{dt} = \dfrac{1}{100}$

(d) $\sec^2\theta \cdot \dfrac{d\theta}{dt} = \dfrac{1}{100}\dfrac{dx}{dt}$

3. $\ln\left(\dfrac{e^x\sqrt{x}}{(5x-1)^{\cos x}}\right) =$

(a) $3x - \frac{1}{2}\ln x - \cos x\ln(5x-1)$

(b) $\dfrac{(5x-1)^{\cos x}}{e^x\sqrt{x}}$

(c) $\dfrac{\ln\left(3x - \frac{1}{2}\ln x\right)}{\ln(\cos x\ln(5x-1))}$

(d) $\ln\left(\dfrac{\frac{1}{2}e^x x^{-1/2}}{-5\sin x(5x-1)^{\cos x}}\right)$

4. If $f(x) = \sqrt[5]{\sin^{-1}(e^x)}$, then $f'(x) =$

(a) $\dfrac{1}{5}(\sin^{-1}(e^x))^{-4/5}$

(b) $\dfrac{e^x(\sin^{-1}(e^x))^{-4/5}}{5\sqrt{1-e^{2x}}}$

(c) $\dfrac{e^x}{5\sqrt{1-e^{2x}}}$

(d) $\dfrac{1}{5}\left(\dfrac{e^x}{\sqrt{1-e^{2x}}}\right)^{-4/5}$

5. The limit $\lim_{x\to 3}(\ln(x-3))^{x^2-9}$ is an indeterminate form of type

(a) ∞^0

(b) 0^0

(c) 1^∞

(d) none; not an indeterminate form

6. $\lim_{x\to 1} x^{\left(\frac{1}{\ln x}\right)} =$

(a) 1

(b) 0

(c) e

(d) does not exist

Fill-In.

1. $\sin^{-1}(1) =$ ____________

2. $\cos^{-1}(1) =$ ____________

3. $\sin^{-1}(\cos 3\pi) =$ ____________

4. $\cos^{-1}\left(\tan\frac{\pi}{4}\right) =$ ____________

5. $\tan\left(\cos^{-1}\frac{2}{7}\right) =$ ____________

6. $\sin\left(\sin^{-1}\frac{5}{2}\right) =$ ____________

7. $\sin^{-1}\left(\sin\frac{3\pi}{2}\right) =$ ____________

Work and Answer. *You must show all relevant work to receive full credit.*

1. If $y\tan y = 3t - \frac{y}{t}$, find $\frac{dy}{dt}$.

2. Find the slope of the tangent line to the graph of $\frac{(x+2)^2}{9} + \frac{y^2}{4} = 1$ at the point $(-2, 2)$.

3. Find the derivative of the function $g(x) = (\sin x)^{3x+1}$.

4. Find the derivative of the function $h(x) = \frac{(3x^2-4)^{10}\cos(4x)}{e^x(59x^3+8x)^{25}}$.

5. A spotlight on the ground shines on a wall 15 m away. If a man 2 m tall walks from the spotlight toward the wall at a speed of 1.8 m/s, how fast is the length of his shadow on the wall decreasing when he is 3 m from the building?

6. Find the limit $\lim_{t \to 0^+} \ln t \sin t$.

7. Find the limit $\lim_{x \to 0^+} (\sin^{-1} x)^x$.

Some kind of **BONUS.**