Math 75A Worksheet $\boxed{\textbf{W2}}$ Name: ______________________

Due with homework on **Friday, October 10**

The Really Awful Truth about $x < 0$

Recall that for $x > 0$ it is true that $\frac{1}{x^3} = \sqrt{\frac{1}{x^6}}$, but for $x < 0$ this is not true! For example, for $x = -1$ we have $\frac{1}{(-1)^3} = -1$, but $\sqrt{\frac{1}{(-1)^6}} = 1$. So for $x < 0$ we have

$$\frac{1}{x^3} = -\sqrt{\frac{1}{x^6}}$$

(notice the extra minus sign).

On the other hand, $\frac{1}{x^3} = \sqrt[3]{\frac{1}{x^9}}$ for *all* $x \neq 0$, both positive and negative! (Check it for $x = -1$ to verify.)

The best way to figure out if you need to add a minus sign in this type of situation is to test it with $x = -1$. If it comes out wrong, put in a minus sign.

Here are some exercises to check your understanding:

Part I.

For each expression in exercises 1 to 7, assume $x < 0$. Decide whether a minus sign should be added to the front of the radical. Put in a (+) or (−) sign for each one to make the statement correct for $x < 0$.

1. $\frac{1}{x} = \quad \sqrt{\frac{1}{x^2}}$
2. $\frac{1}{x^2} = \quad \sqrt{\frac{1}{x^4}}$
3. $\frac{1}{x^5} = \quad \sqrt{\frac{1}{x^{10}}}$
4. $\frac{1}{x^5} = \quad \sqrt[3]{\frac{1}{x^{15}}}$
5. $\frac{1}{x^5} = \quad \sqrt[4]{\frac{1}{x^{20}}}$
6. $\frac{1}{x^{2/3}} = \quad \sqrt[3]{\frac{1}{x^2}}$
7. $\frac{1}{x^{1/5}} = \quad \sqrt[4]{\frac{1}{x^{4/5}}}$

For exercises 8 to 14, fill in the correct power to make the statement true, *and* fill in the correct sign in front of the radical, assuming $x < 0$.

8. $\frac{1}{x^2} = \quad \sqrt{\frac{1}{x^{\square}}}$
9. $\frac{1}{x^3} = \quad \sqrt[4]{\frac{1}{x^{\square}}}$
10. $\frac{1}{x} = \quad \sqrt[3]{\frac{1}{x^{\square}}}$
11. $\frac{1}{x^4} = \quad \sqrt[9]{\frac{1}{x^{\square}}}$
12. $\frac{1}{x^4} = \quad \sqrt[3]{\frac{1}{x^{\square}}}$
13. $\frac{1}{x^{1/3}} = \quad \sqrt[6]{\frac{1}{x^{\square}}}$
14. $\frac{1}{x^{4/7}} = \quad \sqrt{\frac{1}{x^{\square}}}$

over for more fun!

Part II. Find each limit. Be careful when $x \to -\infty$! You may complete these problems on separate paper if you need more room.

1. $\displaystyle\lim_{x\to\infty} \frac{\sqrt{x^2-3x+10}}{x+2}$

2. $\displaystyle\lim_{x\to-\infty} \frac{\sqrt{x^2-3x+10}}{x+2}$

3. $\displaystyle\lim_{x\to-\infty} \frac{-3x^2+4x-2}{\sqrt[6]{8x^{12}-9x^{11}-5x}}$

4. $\displaystyle\lim_{x\to-\infty} \frac{x^4-2x^3+1}{\sqrt{3x^8+5x^6-x+2}}$

5. $\displaystyle\lim_{x\to-\infty} \frac{\sqrt[4]{2x^{12}-3x^6-1}}{x^3+x-5}$

6. $\displaystyle\lim_{x\to-\infty} \frac{-x^3-2x^2+1}{\sqrt[3]{3x^{10}+4x^7-x^2+x}}$

7. $\displaystyle\lim_{x\to-\infty} \frac{\sqrt{x^{2/3}-1}}{3x^2+5x-2}$

8. $\displaystyle\lim_{x\to-\infty} \frac{x+3}{\sqrt[6]{5x^{12/7}-2x^{1/7}-9}}$