Math 75B Midterm I (blue) $/_{100}$ Name: ______________________

Fall 2008

Ch. 9-11, 14-C.2 (Ebersole), §§2.6, 2.7, 3.3, 3.5, 3.7 (Stewart)

Things to remember:

1. Please read directions carefully. Raise your hand if you are not sure what a problem is asking.
2. *You must explain your work thoroughly and unambiguously to receive full credit on questions or parts of questions designated as **Work and Answer**.*
3. **No calculators or notes are allowed on this exam.**
4. You have 50 minutes to complete your test, unless announced otherwise. Do not spend too long on any one problem. You do not have to do the problems in order. Do the easy ones first. Do not attempt the bonus question until you have completed the rest of the test. Before turning in your test, please make sure you have answered and double-checked all the questions.
5. If you need scratch paper, please raise your hand. You may not use your own paper. When you have finished your exam, please turn in any scratch paper you use.
6. For **Work and Answer** problems, write your solutions in the space provided for each problem, or provide specific instructions as to where your work is to be found. *Make it clear what you want and don't want graded.* Your final answers should be boxed or circled.
7. Don't stress! I'm rooting for you!

Multiple Choice. (30 points) *Circle the letter of the best answer.*

1. If y is a function of x, then $\dfrac{d}{dx}(x\sin y) =$

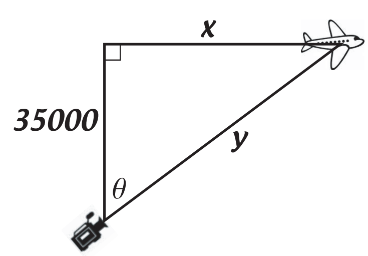
   (a) $\cos y \cdot \dfrac{dy}{dx}$

   (b) $x\cos y \cdot \dfrac{dy}{dx} + \sin y$

   (c) $x\cos y + \sin y$

   (d) $x\cos y \cdot \dfrac{dy}{dx} + \sin y \cdot \dfrac{dy}{dx}$

2. An airplane flies directly over a camera and continues at an altitude of 35,000 ft. The camera stays focused on the plane. According to the picture at right, the quantity $\frac{dy}{dt}$ represents



   (a) the speed of the plane at time t

   (b) the distance from the camera to the plane at time t

   (c) the rate at which the distance from the camera to the plane changes at time t

   (d) the rate at which the angle of the camera is changing at time t

3. If A and r are functions of t and $A^2 = 6\sin r$, then $\dfrac{dA}{dt} =$

(a) $\dfrac{3\cos r}{A}\dfrac{dr}{dt}$

(b) $\dfrac{6\cos r}{A}$

(c) $\dfrac{-3\cos r}{\frac{dA}{dt}}$

(d) $\dfrac{3\cos\left(\frac{dr}{dt}\right)}{A}$

4. If $f(x) = (\tan^{-1}(x))^x$, then $\ln(f(x)) =$

(a) $\ln\left(\dfrac{1}{1+x^2}\right)$

(b) $\dfrac{1}{\tan^{-1}(x)}$

(c) $x\tan^{-1}(x)$

(d) $x\ln(\tan^{-1}(x))$

5. The limit $\lim_{x\to 0}\left(\frac{1}{x^2}\right)^{\cos x - 1}$ represents an indeterminate form of type

(a) ∞^0

(b) 0^0

(c) 1^∞

(d) none; not an indeterminate form

6. According to l'Hôpital's Rule, $\lim_{x\to\infty}\dfrac{3x^2}{\ln x} =$

(a) $\lim_{x\to\infty}\dfrac{6x}{\ln x}$

(b) $\lim_{x\to\infty}\dfrac{6x\ln x - 3x}{(\ln x)^2}$

(c) $\lim_{x\to\infty}\dfrac{6x}{\frac{1}{x}}$

(d) $\lim_{x\to 0}\dfrac{3x^2}{\ln x}$

Fill-In. (30 points)

1. Compute each quantity. If the answer is an angle, it must be in radians. If there is no correct answer, write "undefined."

$\sin^{-1}(1)$		$\tan^{-1}\left(\sin\left(\frac{\pi}{2}\right)\right)$		$\sin\left(\cos^{-1}\left(\frac{5}{6}\right)\right)$	
$\cos^{-1}\left(-\frac{1}{2}\right)$		$\sin^{-1}(\cos(0))$		$\cos\left(\cos^{-1}\left(\frac{11}{2}\right)\right)$	
$\tan^{-1}(1)$		$\cos^{-1}\left(\sec\left(\frac{\pi}{3}\right)\right)$		$\cos\left(\tan^{-1}\left(-\sqrt{3}\right)\right)$	

2. For each function, fill in the derivative.

$f(x)$	$f'(x)$	$f(x)$	$f'(x)$
$2\cos^{-1}(x)$		$\tan^{-1}(8x)$	
$x\cos^{-1}(x)$		$\sin^{-1}(\sqrt{x})$	

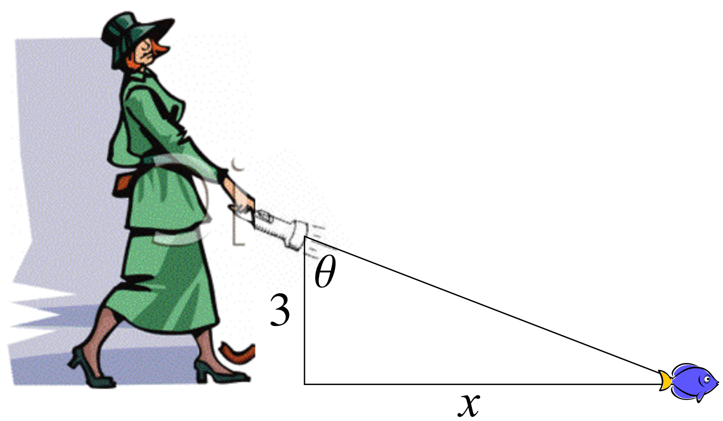
Work and Answer. (40 points) *You must show all relevant work to receive full credit. Be sure to box or circle your final answers. In word problems, be sure to give units (e.g. feet, kilograms, etc.).*

1. If $\sin y = 3x^2 + y$, find $\dfrac{dy}{dx}$.

2. Find the derivative of the function $g(x) = \dfrac{x^{10}\sin x}{(3x^2+5)^{12}}$.

Hint. *Logarithmic differentiation may be helpful.*

3. A woman walks at a speed of 2 ft./s with a flashlight held 3 ft. above the ground. Her flashlight remains focused on a stationary object along her path, as shown. When she is 6 ft. from the object, how fast is the angle θ decreasing? *Be sure to give units on your answer.*



4. Compute the limit $\lim_{x \to \infty} \frac{\ln x}{\sqrt[3]{x}}$.

BONUS. (5 points) Find the limit $\lim_{x \to \infty} x(\ln(x+7) - \ln(x))$.