Math 75B Midterm II (blue) $/_{100}$ Name: ____________________

Fall 2008

Ch. 16, 17, 12 (E), §§4.1-4.5, 2.8 (S)

Things to remember:

1. Please read directions carefully. Raise your hand if you are not sure what a problem is asking.
2. *You must explain your work thoroughly and unambiguously to receive full credit on questions or parts of questions designated as* ***Work and Answer****.*
3. **No calculators or notes are allowed on this exam.**
4. You have 65 minutes to complete your test, unless announced otherwise. Do not spend too long on any one problem. You do not have to do the problems in order. Do the easy ones first. Do not attempt the bonus question until you have completed the rest of the test. Before turning in your test, please make sure you have answered and double-checked all the questions.
5. If you need scratch paper, please raise your hand. You may not use your own paper. When you have finished your exam, please turn in any scratch paper you use.
6. Write your solutions in the space provided for each problem, or provide specific instructions as to where your work is to be found. *Make it clear what you want and don't want graded.* Your final answers should be boxed or circled.
7. Unless directed otherwise, only EXACT ANSWERS will receive full credit (i.e. $\sqrt{2}$, not 1.414).
8. In word problems, give units on all answers (e.g. feet, grams, gallons).
9. Don't stress! I'm rooting for you!

True or False. (15 points) Circle **T** if the statement is *always* true; otherwise circle **F**.

1. The absolute minimum value of $f(x) = x^2 - 4x + 7$ is 2. **T** **F**

2. The function $f(x) = \dfrac{x^4}{\cos x}$ is an even function. **T** **F**

3. Let $f(x) = x \ln x - 2$. According to the Intermediate Value Theorem, there is at least one root of $f(x)$ between 1 and e. **T** **F**

4. Let $f(x) = x \ln x - 2$. Then $f'(x) = 1 + \ln x$. According to Rolle's Theorem, $f(x)$ has at least 2 real roots. **T** **F**

5. The differential of the function $y = \ln x - 3x + 4$ at $x = 1$ with $dx = 0.1$ is $dy = -0.2$. **T** **F**

Multiple Choice. (25 points) *Circle the letter of the best answer.*

1. A farmer has 4500 meters of fencing with which to build three sides of a rectangular pigpen of largest possible area. This problem is solved by maximizing the equation

   (a) $2l + w = 4500$ (c) $A = 4500l - 2l^2$

   (b) $A = 2l + w$ (d) $4500 = lw$

2. The function $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x$, whose derivative is $f'(x) = x^2 + x - 2$, has critical numbers

   (a) -2 and 1 (c) -1 only

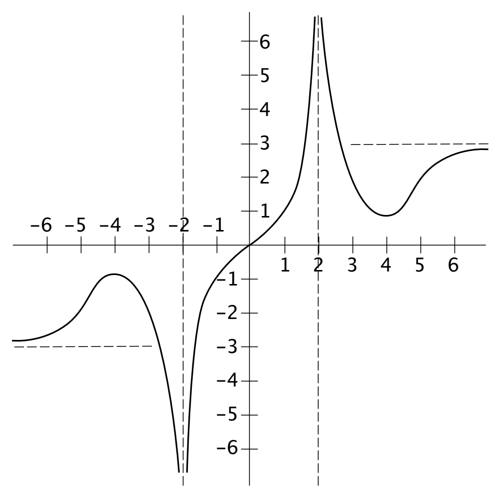
   (b) 2 and 0 (d) none; $f(x)$ has no critical numbers

3. From the graph of $f(x)$ shown, $f(x)$

   (a) is an odd function

   (b) is an even function

   (c) is neither an odd nor an even function



4. The linear approximation of the function $f(x) = x^3$ at $x = 2$ is

   (a) $y = 12x - 16$ (c) $y = 6x - 8$

   (b) $y = 3x - 2$ (d) $y = 3x^2$

5. The function $f(x) = x^3 + 5x - 1$ has a root

   (a) between 0 and $\frac{1}{2}$ (c) between 1 and $\frac{3}{2}$

   (b) between $\frac{1}{2}$ and 1 (d) between $\frac{3}{2}$ and 2

Fill-In. (12 points) *If there is no answer to a question, write 'NONE.'*

1. The function $f(x) = x^3 - 2x^2 + 5$ is increasing on the interval(s) __________________

   and decreasing on the interval(s) __________________ .

2. The function $f(x) = x^3 - 2x^2 + 5$ is concave up on the interval(s) __________________

   and concave down on the interval(s) __________________ .

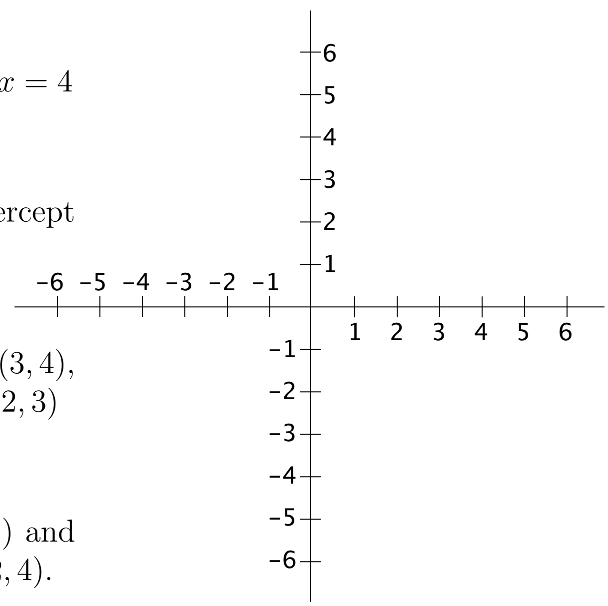
Graph. (12 points)
On the axes below, sketch the graph of a function $f(x)$ satisfying all of the following:

(a) The domain of $f(x)$ is $x \neq 2$, $x \neq 4$

(b) $f(x)$ has vertical asymptotes at $x = 2$ and $x = 4$

(c) $f(x)$ has a horizontal asymptote $y = -1$

(d) $f(x)$ has y-intercept $(0, 1)$ and one x-intercept $(-1, 0)$



(e) $f(x)$ is increasing on the intervals $(-\infty, 2)$, $(3, 4)$, and $(4, \infty)$ and decreasing on the interval $(2, 3)$

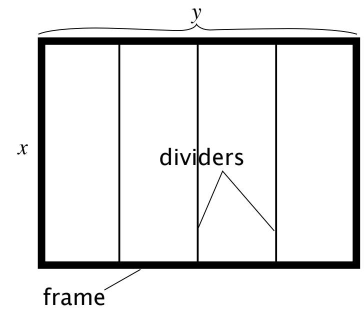
(f) $f(x)$ has a critical point $(3, 1)$

(g) $f(x)$ is concave down on the interval $(4, \infty)$ and concave up on the intervals $(-\infty, 2)$ and $(2, 4)$.

Work and Answer. (36 points) *You must show all relevant work to receive full credit. Be sure to box or circle your final answers.*

1. Find the x-value where the function $g(x) = x^3 + 3x^2 - 9x + 1$ attains its absolute minimum on the interval $[0, 2]$.

2. A contractor wishes to build a rectangular window with four panes, as shown. The material for the **frame** of the window costs \$10 per foot, while the material for the dividers between the windows costs \$5 per foot. If the window must have an area of 70 ft.^{2}, what dimensions will minimize the cost of the window? *(You do not have to account for the cost of the glass.)*



3. Use a linear approximation to estimate $\sqrt{83}$. *You may use differentials if you prefer.*

BONUS. (5 points)

If the derivative of $f(x)$ is $f'(x) = \dfrac{x^2 - 5}{x - 3}$, find the x-coordinate(s) of the inflection point(s) of $f(x)$.