Math 75B Selected Homework Solutions

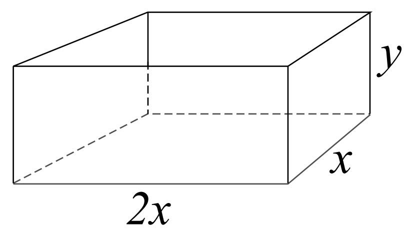
16-B #1, 2, 4
4.5 #2, 12, 21, 32, 43*

Completeness:	14	(2 points each)
Format:	10	
Total:	24	points
	(+2	possible bonus points)

---

§4.5 #12. A box with no top, whose length is twice its width, is to have a volume of 10 m^3. It is to be made from material costing \$10 per square meter for the base, and \$6 per square meter for the sides. What is the minimum cost?

The objective of the problem is to **minimize the cost**, so we must write a formula for the **cost** of the box. First we assign names to the dimensions of the box, as shown. The problem says that the length of the base is twice the width, so we let x be the width and then the length is $2x$.



The area of the base is $2x \cdot x = 2x^2$, so the cost of the base is $10 \cdot 2x^2 = 20x^2$ (dollars). Similarly, the area of the sides is $2xy + 2(2x \cdot y) = 6xy$, so the cost of the sides is $6 \cdot 6xy = 36xy$. Therefore the cost of the entire box, in dollars, is

$$C = 20x^2 + 36xy.$$

We know that the volume of the box is 10 m^2, so we have

$$2x \cdot x \cdot y = 10$$
$$2x^2 y = 10$$
$$y = \frac{10}{2x^2} = \frac{5}{x^2}.$$

Plugging this into the cost formula, we get

$$C = 20x^2 + 36x\left(\frac{5}{x^2}\right)$$
$$= 20x^2 + \frac{180}{x}.$$

Now we can take the derivative in order to find the maximum value of C. We have

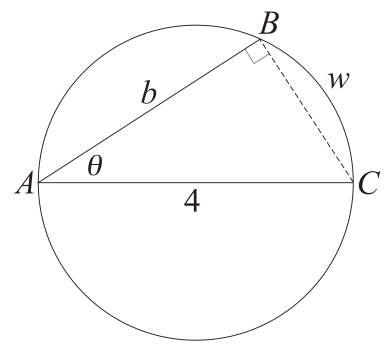
$$C'(x) = 40x - \frac{180}{x^2} \overset{\text{set}}{=} 0$$
$$40x = \frac{180}{x^2}$$
$$40x^3 = 180$$
$$x^3 = \frac{180}{40} = \frac{9}{2}$$
$$x = \sqrt[3]{\frac{9}{2}}.$$

You can check that this represents the x-value that gives the absolute minimum of the function $C(x)$.

Rereading the problem, we see that the problem asks us for the minimum cost. So

$$C\left(\sqrt[3]{\frac{9}{2}}\right) = 20\left(\sqrt[3]{\frac{9}{2}}\right)^2 + \frac{180}{\sqrt[3]{\frac{9}{2}}} \approx \boxed{\$163.54}$$

§4.5 #32. A woman at point A of a circular lake with radius 2 miles wants to arrive at point C (see figure) in the shortest possible time. She can walk at the rater of 4 mi./hr. and row a boat at 2 mi./hr. How should she proceed?



The objective of the problem is to **minimize the time**. The time she will spend getting from point A to point C is

$$\begin{aligned} T &= (\text{time spent rowing}) + (\text{time spent walking}) \\ &= \frac{b}{2} + \frac{w}{4} \end{aligned}$$

since time is equal to distance divided by (constant) velocity.

Using the geometry facts I noted on the Homework List (scroll down to the bottom), we can express both b and w as functions of the angle θ, where $\theta = 0$ represents the path of rowing straight across the lake (and not doing any walking), $\theta = \frac{\pi}{2}$ represents the path of walking around the lake (and not rowing at all), and the angles in between represent the paths of rowing to a point B and walking the rest of the way.

We have $\cos\theta = \frac{b}{4}$, so $b = 4\cos\theta$. Also $w = 2r\theta = 4\theta$. Therefore our formula for time looks like

$$\begin{aligned} T &= \frac{4\cos\theta}{2} + \frac{4\theta}{4} \\ &= 2\cos\theta + \theta. \end{aligned}$$

Now we take the derivative to find the critical numbers. We have

$$\begin{aligned} T'(\theta) = -2\sin\theta + 1 &\overset{\text{set}}{=} 0 \\ \sin\theta &= \frac{1}{2} \\ \theta &= \frac{\pi}{6} \end{aligned}$$

(since $0 \leq \theta \leq \frac{\pi}{2}$ as noted above).

Is this how she should proceed, then? Row at an angle of $\frac{\pi}{6}$ and then walk the rest of the way? We'd better check the endpoints as well! We know that the time will be minimized at one of the following angles: 0, $\frac{\pi}{6}$, or $\frac{\pi}{2}$. Let us check each one:

$$\begin{aligned} T(0) &= 2\cos(0) + 0 = 2 \text{ hours} \\ T\left(\frac{\pi}{6}\right) &= 2\cos\left(\frac{\pi}{6}\right) + \frac{\pi}{6} = 2 \cdot \frac{\sqrt{3}}{2} + \frac{\pi}{6} \approx 2.26 \text{ hours} \\ T\left(\frac{\pi}{2}\right) &= 2\cos\left(\frac{\pi}{2}\right) + \frac{\pi}{2} = 0 + \frac{\pi}{2} \approx 1.57 \text{ hours} \end{aligned}$$

Whoops! It would actually take the *longest* to row at $\frac{\pi}{6}$! The path that will **minimize** the time is for her to $\boxed{\text{walk the whole way}}$