Math 75B Selected Homework Solutions

Completeness:	26	
Format:	10	
Total:	36	points

18-A #2, 4
18-B #1, 3
4.7 #2, 6, 10, 19, 28, 44, 46
19-A #3, 4
20-A #4
20-B #1, 2, 4, 5
5.2 #2, 11, 16, 33, 36, 38, 39, 40

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§4.7 #28. If $f''(t) = 2e^t + 3\sin t$ and $f(0) = 0$ and $f(\pi) = 0$, find $f(t)$.

First, we have $f'(t) = 2e^t - 3\cos t + C$; therefore $f(t) = 2e^t - 3\sin t + Ct + D$, where C and D are constants. To find out what C and D are, we use the facts provided. We have

$$0 = f(0) = 2e^0 - 3\sin(0) + C(0) + D = 2 + D,$$

so $D = -2$. Also

$$0 = f(\pi) = 2e^\pi - 3\sin(\pi) + C(\pi) + (-2)$$

(since we now know that $D = -2$)

$$= 2e^\pi - \pi C - 2.$$

Solving for C we get

$$\pi C = 2e^\pi - 2$$
$$C = \frac{2e^\pi - 2}{\pi}.$$

Therefore we have

$$f(t) = \boxed{2e^t - 3\sin t + \frac{(2e^\pi - 2)t}{\pi} - 2}$$

§4.7 #44. A car is traveling at 50 mi./hr. when the brakes are fully applied, producing a constant deceleration of 22 ft./s^2. What is the distance traveled before the car comes to a stop?

This is a tricky problem. The first tricky part is that the units on the numbers are not the same! We will convert 50 mi./hr. to feet per second to make it match the units on the other number. We have

$$\frac{50 \text{ mi.}}{1 \text{ hr.}} \cdot \frac{5280 \text{ ft.}}{1 \text{ mi.}} \cdot \frac{1 \text{ hr.}}{3600 \text{ s}} = \frac{50 \cdot 5280}{3600} = 73.\overline{3} \text{ ft./s.}$$

For this problem let $t = 0$ denote the moment the brakes are applied. We wish to find the distance traveled by the car starting from that point. So we know the following:

$$v(0) = 73.\overline{3} \tag{1}$$
$$s(0) = 0 \tag{2}$$

where v stands for velocity and s stands for distance.

Recall that distance, velocity, and acceleration are all related via derivatives/antiderivatives as follows:

$$\begin{gathered}\text{distance}\\ \downarrow\uparrow\\ \text{velocity}\\ \downarrow\uparrow\\ \text{acceleration}\end{gathered}$$

where the down arrow $\downarrow$ means "derivative" and the up arrow $\uparrow$ means "antiderivative." For example, **velocity** is an *antiderivative* of **acceleration**.

We are given that

$$a(t) = -22.$$

Therefore the velocity at time t is $v(t) = \int a(t)\, dt = \int (-22)\, dt = -22t + C$. By (1) we have $73.\overline{3} = v(0) = -22(0) + C = C$. Thus the velocity formula is now

$$v(t) = -22t + 73.\overline{3}.$$

Now we take one more antiderivative to get the distance. We have $s(t) = \int v(t)\ dt = -11t^2 + 73.\overline{3}t + D$. Moreover, by (2) we know that $0 = s(0) = 0 + D$, so $D = 0$ and

$$s(t) = \int v(t)\ dt = -11t^2 + 73.\overline{3}t.$$

If we can figure out *how long* the car was braking, we can plug this time into $s(t)$ to get the distance traveled by the car during the skid. Observe that, at the end of the skid (when the car comes to a stop), the velocity is 0. So we know

$$v(t_f) = -22t_f + 73.\overline{3} = 0$$

where t_f (read as "t-final") is the time at the end of the skid. Solving for t_f we get $t_f = \dfrac{73.\overline{3}}{22}$. Therefore the distance traveled by the car during the skid is

$$s(t_f) = -11\left(\frac{73.\overline{3}}{22}\right)^2 + 73.\overline{3}\cdot\frac{73.\overline{3}}{22} = -\frac{(73.\overline{3})^2}{2\cdot 22} + \frac{(73.\overline{3})^2}{22} = \frac{(73.\overline{3})^2}{44}(-1+2) = \boxed{122.\overline{2}\text{ ft.}}$$

§4.7 #46. A car braked with a constant deceleration of 16 ft./s^2, producing skid marks measuring 200 ft. before coming to a stop. How fast was the car traveling when the brakes were first applied?

This problem is similar to #44, above, but it is even trickier! This time we know that

$$v(t_f) = 0 \tag{3}$$
$$s(0) = 0 \tag{4}$$
$$s(t_f) = 200 \tag{5}$$

and we are supposed to find $v(0)$. We have $a(t) = -16$, so $v(t) = -16t + C$ and $v(0) = C$. Therefore the problem will be solved by finding C.

We have $s(t) = \int v(t)\, dt = -8t^2 + Ct + D$; therefore by (4) $s(0) = D = 0$, and $s(t) = -8t^2 + Ct$. By (5) and (3), respectively, we have

$$s(t_f) = -8(t_f)^2 + Ct_f = 200 \tag{6}$$
$$v(t_f) = -16t_f + C = 0$$

Solving the second equation for t_f we get $t_f = \dfrac{C}{16}$, which we may then plug into (6) to get

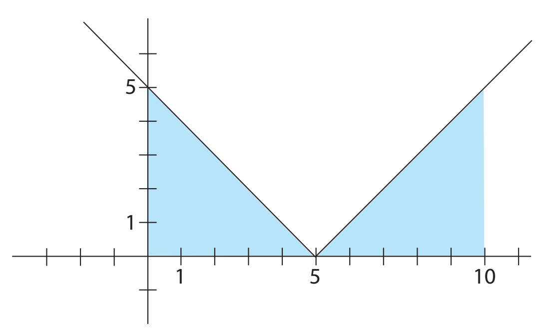
$$\begin{aligned} -8\left(\frac{C}{16}\right)^2 + C\left(\frac{C}{16}\right) &= 200 \\ \frac{-8C^2}{16 \cdot 16} + \frac{C^2}{16} &= 200 \\ \frac{-C^2 + 2C^2}{2 \cdot 16} &= 200 \\ C^2 = 200 \cdot 32 &= 100 \cdot 64 \\ C = 10 \cdot 8 &= 80 \end{aligned}$$

So the car was going $\boxed{80 \text{ ft./s } (= 54.\overline{54} \text{ mi./hr.})}$ when the brakes were applied.

§5.2 #36. Use geometry to evaluate the integral $\displaystyle\int_0^{10} |x - 5|\, dx$.

The area to be found is as shown at right.
We have two triangles with base 5 and height 5, so the total area is

$$\frac{5 \cdot 5}{2} + \frac{5 \cdot 5}{2} = 25.$$



Since the regions lie entirely above the x-axis, all of the area counts positively toward the definite integral. Therefore $\displaystyle\int_0^{10} |x - 5|\, dx = \boxed{25}$

§5.2 #38. Evaluate the integral $\int_1^1 x^2 \cos x \, dx$.

Since 1 is in the domain of the integrand $x^2 \cos x$, by the properties of definite integrals we have $\int_1^1 x^2 \cos x \, dx = \boxed{0}$ (see the notes from class or the bottom of page 269 of Stewart for more details).

§5.2 #40. If $\int_1^5 f(x) \, dx = 12$ and $\int_4^5 f(x) \, dx = 3.6$, find $\int_1^4 f(x) \, dx$.

By the properties of integrals, we know that $\int_1^4 f(x) \, dx + \int_4^5 f(x) \, dx = \int_1^5 f(x) \, dx$. Therefore we have $\int_1^4 f(x) \, dx + 3.6 = 12$, so $\int_1^4 f(x) \, dx = 12 - 3.6 = \boxed{8.4}$