Math 75B Bonus Assignment $/_{10}$ Name: ______________________

Reversing the Chain Rule (u-substitution)

Due at the end of class on **Monday, November 24**

How might we solve the integral $\int x(x^2-5)^3\,dx$?

We can try guessing and checking, as we did in class. Or, we can try a more systematic approach:

In the integrand we have to look for a "chunk" and for the derivative of the chunk. In this case a good chunk to use is x^2-5, especially since the derivative, $2x$, is *almost* in the integrand (all except the 2). Let $u=x^2-5$. Then $\frac{du}{dx}=2x$. Now "multiply both sides by dx" to get $du=2x\,dx$. You can either solve this equation for $x\,dx=\frac{1}{2}\,du$ and substitute it directly into the integral, or you can "futz" the 2 in the integral first, whichever you prefer. The latter process looks like

$$\int x(x^2-5)^3\,dx=\frac{1}{2}\int 2x(x^2-5)^3\,dx=\frac{1}{2}u^3\,du$$

(you should get the same thing if you use the "solve for what you want" method). This is a much easier integral to do! We have

$$\frac{1}{2}u^3\,du=\frac{1}{2}\cdot\frac{1}{4}u^4+C=\frac{1}{8}u^4+C.$$

Now just "back-substitute" to get the answer in terms of x:

$$\frac{1}{8}u^4+C=\boxed{\frac{1}{8}(x^2-5)^4+C}$$

Now try these: evaluate each integral by making a u-substitution. Check by differentiating.

1. $\int x^2(5x^3+8)^7\,dx$

$u=$ ____________________

$du=$ ____________________ dx

over for more fun!

2. $\int \frac{\ln x}{x}\, dx$

$u =$ ______________________

$du =$ ______________________ dx

3. $\int x \sec(x^2) \tan(x^2)\, dx$

$u =$ ______________________

$du =$ ______________________ dx

4. $\int (\cos x) \sqrt[3]{\sin x}\, dx$

$u =$ ______________________

$du =$ ______________________ dx

5. $\int (\sin x) e^{\cos x + 1}\, dx$

$u =$ ______________________

$du =$ ______________________ dx