Math 75B Bonus Assignment – Solutions
Reversing the Chain Rule (u-substitution)

Evaluate each integral by making a u-substitution. Check by differentiating.

1. $\displaystyle\int x^2(5x^3+8)^7\,dx$

$u = \underline{\quad 5x^3+8 \quad}$

$du = \underline{\quad 15x^2 \quad}\ dx$

We have

$$\begin{aligned}\int x^2(5x^3+8)^7\,dx &= \frac{1}{15}\int 15x^2(5x^3+8)^7\,dx \\ &= \frac{1}{15}\int u^7\,du \\ &= \frac{1}{15}\cdot\frac{1}{8}u^8 + C \\ &= \boxed{\frac{1}{120}(5x^3+8)^8 + C}\end{aligned}$$

2. $\displaystyle\int \frac{\ln x}{x}\,dx$

$u = \underline{\quad \ln x \quad}$

$du = \underline{\quad \frac{1}{x} \quad}\ dx$

We have

$$\begin{aligned}\int \frac{\ln x}{x}\,dx &= \int \ln x\cdot\frac{1}{x}\,dx \\ &= \int u\,du \\ &= \frac{1}{2}u^2 + C \\ &= \boxed{\frac{1}{2}(\ln x)^2 + C}\end{aligned}$$

3. $\displaystyle\int x\sec(x^2)\tan(x^2)\,dx$

$u = \underline{\quad x^2 \quad}$

$du = \underline{\quad 2x \quad}\ dx$

We have

$$\begin{aligned}\int x\sec(x^2)\tan(x^2)\,dx &= \frac{1}{2}\int 2x\sec(x^2)\tan(x^2)\,dx \\ &= \frac{1}{2}\int \sec u\tan u\,du \\ &= \frac{1}{2}\sec u + C \\ &= \boxed{\frac{1}{2}\sec(x^2) + C}\end{aligned}$$

4. $\int (\cos x) \sqrt[3]{\sin x}\ dx$

$u = \underline{\quad \sin x \quad}$

$du = \underline{\quad \cos x \quad}\ dx$

We have

$$\begin{aligned}
\int (\cos x) \sqrt[3]{\sin x} &= \int \sqrt[3]{u}\ du \\
&= \int u^{1/3}\ du \\
&= \frac{3}{4} u^{4/3} + C \\
&= \boxed{\frac{3}{4} (\sin x)^{4/3} + C}
\end{aligned}$$

5. $\int (\sin x) e^{\cos x + 1}\ dx$

$u = \underline{\quad \cos x + 1 \quad}$

$du = \underline{\quad -\sin x \quad}\ dx$

We have

$$\begin{aligned}
\int (\sin x) e^{\cos x + 1}\ dx &= -\int (-\sin x) e^{\cos x + 1}\ dx \\
&= -\int e^u\ du \\
&= -e^u + C \\
&= \boxed{-e^{\cos x + 1} + C}
\end{aligned}$$