Math 76 Practice Problems for Midterm II
§§6.4-8.1

DISCLAIMER. This collection of practice problems is *not* guaranteed to be identical, in length or content, to the actual exam. You may expect to see problems on the test that are not exactly like problems you have seen before.

On the actual exam you will have more room to work the problems. You will see directions similar to these:

1. Please read directions carefully. Raise your hand if you are not sure what a problem is asking.
2. *You must explain your work thoroughly and unambiguously to receive full credit on questions or parts of questions designated as* ***Work and Answer****.*
3. **No calculators or notes are allowed on this exam. All electronic devices must be stowed and silent.**
4. You have 65 minutes to complete your test, unless announced otherwise. Do not spend too long on any one problem. You do not have to do the problems in order. Do the easy ones first. Do not attempt the bonus question until you have completed the rest of the test. Before turning in your test, please make sure you have answered and double-checked all the questions.
5. If you need scratch paper, please raise your hand. You may not use your own paper. When you have finished your exam, please turn in any scratch paper you use.
6. Write your solutions in the space provided for each problem, or provide specific instructions as to where your work is to be found. *Make it clear what you want and don't want graded.* Your final answers should be boxed or circled.
7. Unless directed otherwise, only EXACT ANSWERS will receive full credit (i.e. $\sqrt{2}$, not 1.414).
8. In word problems, give units on all answers (e.g. feet, grams, gallons).
9. Don't stress! I'm rooting for you!

You will also be provided a page of integral tables (on the overhead) and the following information:

English system formulas:	Metric system formulas:
1 ft. = 12 in.	$F = m \cdot a$
5280 ft. = 1 mi.	$g = 9.8\,\mathrm{m/s^2}$
16 oz. = 1 lb.	100 cm = 1 m
Weight of water: $\omega = 62.5\,\mathrm{lb./ft^3}$	Weight of water: $\omega = 9800\,\mathrm{N/m^3}$

General formulas:

Hooke's Law: $F(x) = kx$

$$W = \omega \int_0^b (x + P)A(x)\, dx$$

$$H.F. = \omega \int_a^b w(y)d(y)\, dy$$

$$\overline{x} = \frac{1}{A}\int_a^b x\,(f(x) - g(x))\, dx \qquad \overline{y} = \frac{1}{A}\cdot\frac{1}{2}\int_a^b \left(f(x)\right)^2 - \left(g(x)\right)^2)\, dx$$

Multiple Choice. *Circle the letter of the best answer.*

1. $\displaystyle\int_0^e \ln x \, dx =$

   (a) 1 (c) ∞

   (b) 0 (d) $-\infty$

2. What expression best represents the area between $x = y^2$ and $x = -y$ from $y = -1$ to $y = 1$?

   (a) $\displaystyle\int_{-1}^{0} (y^2 + y)\, dy + \int_0^1 (-y - y^2)\, dy$

   (b) $\displaystyle\int_{-1}^{0} (-y - y^2)\, dy + \int_0^1 (y^2 + y)\, dy$

   (c) $\displaystyle\int_{-1}^{1} (y^2 + y)\, dy$

   (d) $\displaystyle\int_{-1}^{0} (y^2 - y)\, dy + \int_0^1 (y - y^2)\, dy$

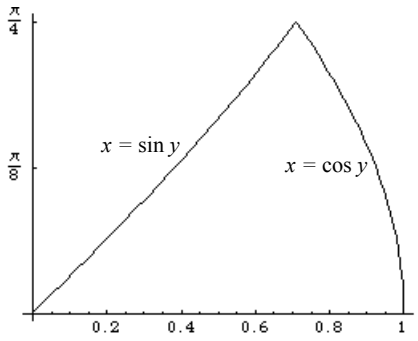
3. The volume of the solid formed by rotating the region shown about the y-axis is

   (a) $\displaystyle 2\pi \int_0^{\pi/4} y\,(\sin y - \cos y)\; dy$

   (b) $\displaystyle \pi \int_0^{\pi/4} (\cos y - \sin y)^2\; dy$

   (c) $\displaystyle \pi \int_0^{\pi/4} \left(\cos^2 y - \sin^2 y\right)\; dy$

   (d) $\displaystyle 2\pi \int_0^{\pi/4} y\,(\cos y - \sin y)\; dy$



4. The volume of the solid formed by rotating the region enclosed by the curves $y = \dfrac{1}{x^3}$, $y = \dfrac{1}{x^2}$, and $x = 2$ about the line $x = -1$ is

   (a) $\displaystyle 2\pi \int_0^2 (x+1)\left(\frac{1}{x^3} - \frac{1}{x^2}\right) dx$ (c) $\displaystyle 2\pi \int_0^2 (x-1)\left(\frac{1}{x^2} - \frac{1}{x^3}\right) dx$

   (b) $\displaystyle 2\pi \int_1^2 (x+1)\left(\frac{1}{x^2} - \frac{1}{x^3}\right) dx$ (d) $\displaystyle 2\pi \int_1^2 (1-x)\left(\frac{1}{x^2} - \frac{1}{x^3}\right) dx$

5. Lois Lane, whose mass is 50 kg, is hanging from a 20-meter rope tied to a crane. Superman is at the top of the crane. In order to rescue Lois, he must pull the rope all the way up to the top of the crane. If the rope has a mass of 10 kg, then the work Superman must do in order to rescue Lois is

   (a) 10,780 N (c) 9,800 N

   (b) 10,780 J (d) 9,800 J

6. A rectangular aquarium 4 ft. wide, 6 ft. long, and 2 ft. high is full of water. If a pump is placed at the top of the tank, the work done in pumping *half* the water out is

   (a) 62.5(6) ft.-lb.
   (b) 62.5(12) ft.-lb.
   (c) 62.5(24) ft.-lb.
   (d) 62.5(48) ft.-lb.

7. The length of the curve $x = y^3 - y$ from $y = 1$ to $y = 3$ is

   (a) $2\pi \displaystyle\int_1^3 \sqrt{1 + (3y^2 - 1)^2}\, dy$
   (b) $\displaystyle\int_1^3 \sqrt{9y^4 - 6y^2 + 2}\, dy$
   (c) $\displaystyle\int_1^3 \sqrt{1 + y^3 + y}\, dy$
   (d) $\displaystyle\int_1^3 \sqrt{3y^2}\, dy$

8. A trough is filled with water. The ends of the trough are equilateral triangles with sides 8 m long and vertex at the bottom. The hydrostatic force on one end of the trough is

   (a) $\dfrac{9800\sqrt{3}}{2} \displaystyle\int_0^{4\sqrt{3}} y(y - 8)\, dy$
   (b) $\dfrac{9800}{\sqrt{3}} \displaystyle\int_0^{4\sqrt{3}} y^2\, dy$
   (c) $9800 \displaystyle\int_0^8 (8 - y)y\, dy$
   (d) $\dfrac{19600}{\sqrt{3}} \displaystyle\int_0^{4\sqrt{3}} (4\sqrt{3} - y)y\, dy$

9. The coordinates of the center of mass of the region enclosed by $y = x$, $y = x - 4$, $x = 1$ and $x = 3$ are

   (a) $(2.5, 0.5)$
   (b) $(1.75, 0)$
   (c) $(2, 0)$
   (d) $(2, 0.5)$

10. The nth term of the sequence $\{-3, 4, 11, 18, 25, \ldots\}$, counting $a_1 = -3$ as the first term, is

    (a) $a_n = 5n - 2$
    (b) $a_n = 7n - 10$
    (c) $a_n = n^2 - 4$
    (d) $a_n = -3n + 7$

Fill-In.

1. $\displaystyle\int_1^\infty \frac{5}{x^3}\, dx =$ _____ .

2. If the region enclosed by the curves $y = \sqrt{x + 2}$, $y = 1$ and $x = 2$ is rotated about the x-axis, the volume of the resulting solid is _____ .

3. If the region enclosed by the curves $y = 5 - x^2$ and $y = x + 3$ is rotated about the line $y = 1$, the volume of the resulting solid is _____ .

4. If 25 N of force are required to keep a spring stretched 20 cm beyond its natural length, then the spring constant for the spring is $k =$ _____ .

Work and Answer. *You must show all relevant work to receive full credit.*

1. Find the area enclosed by the curves $y = \dfrac{1}{x^2}$, $y = \dfrac{1}{x^3}$, and $x = 2$.

2. (a) Use the disk method to find the volume of the solid formed by rotating the region enclosed by the curves $x = y^2$ and $x = 2y$ about the line $y = -1$.

   (b) Use the shell method to find the volume of the solid formed by rotating the region enclosed by the curves $x = y^2$ and $x = 2y$ about the line $y = -1$.

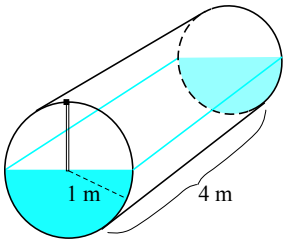
   (c) Should the answers to (a) and (b) be the same? Why or why not?

3. A certain spring has a natural length of 18 in. If 10 lb. of force is needed to keep the spring stretched to a length of 24 in., what is the work done in stretching it to 36 in.?

4. A tank in the shape of a cylinder on its side is half full of water. A pump is at the top of the tank, as shown below.

   (a) Find the hydrostatic force on one of the circular sides of the tank.

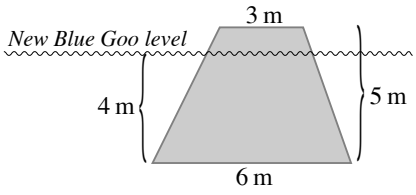
   (b) Set up, **but do not evaluate,** an integral for the work done in pumping all the water out of the tank.



5. Find the length of the curve $f(x) = \dfrac{e^x + e^{-x}}{2}$ from $x = 0$ to $x = 1$.

6. Find the length of the curve $x = \dfrac{y^2 - 1}{2}$ from $y = 1$ to $y = 3$.

7. Find the hydrostatic force on the wall shown. The fluid is New Blue Goo (density 1500 kg/m^3).



8. For the sequence $\{2, -\dfrac{4}{3}, \dfrac{8}{9}, -\dfrac{16}{27}, \ldots\}$,

   (a) Find a formula for the n-th term a_n of the sequence, assuming $a_0 = 2$.

   (b) *Circle the best answer.* The sequence $\{a_n\}_{n=0}^{\infty}$ above **(converges | diverges)**.

Some kind of **BONUS.**