**Math 76 Practice Problems for Midterm III**
§§8.2-9.2

**DISCLAIMER.** This collection of practice problems is *not* guaranteed to be identical, in length or content, to the actual exam. You may expect to see problems on the test that are not exactly like problems you have seen before.

On the actual exam you will have more room to work the problems. You will see directions similar to these:

1. Please read directions carefully. Raise your hand if you are not sure what a problem is asking.
2. *You must explain your work thoroughly and unambiguously to receive full credit on questions or parts of questions designated as* ***Work and Answer****.*
3. **No calculators or notes are allowed on this exam. All electronic devices must be stowed and silent.**
4. You have 65 minutes to complete your test, unless announced otherwise. Do not spend too long on any one problem. You do not have to do the problems in order. Do the easy ones first. Do not attempt the bonus question until you have completed the rest of the test. Before turning in your test, please make sure you have answered and double-checked all the questions.
5. If you need scratch paper, please raise your hand. You may not use your own paper. When you have finished your exam, please turn in any scratch paper you use.
6. Write your solutions in the space provided for each problem, or provide specific instructions as to where your work is to be found. *Make it clear what you want and don't want graded.* Your final answers should be boxed or circled.
7. Unless directed otherwise, only EXACT ANSWERS will receive full credit (i.e. $\sqrt{2}$, not 1.414).
8. Don't stress! I'm rooting for you!

**Multiple Choice.** *Circle the letter of the best answer.*

1. $\displaystyle\sum_{n=1}^{\infty} 3\left(\tfrac{1}{2}\right)^n =$

   (a) 6
   (b) 3
   (c) $\dfrac{3}{2}$
   (d) $\infty$ (diverges)

2. The series $\displaystyle\sum_{n=1}^{\infty} \frac{2}{3^{n+2}}$

   (a) converges to $\dfrac{8}{9}$
   (b) converges to $\dfrac{1}{9}$
   (c) converges to 3
   (d) converges to 9

3. $\displaystyle\sum_{n=3}^{\infty}\left(\frac{2}{n}-\frac{2}{n+1}\right)=$

(a) 0

(b) $\frac{1}{6}$

(c) $\frac{2}{3}$

(d) $\infty$ (diverges)

4. To determine whether or not the series $\displaystyle\sum_{n=2}^{\infty}\frac{5n^3}{1-2n+n^4}$ converges, the limit comparison test may be used with comparison series $\sum b_n =$

(a) $\sum \frac{1}{n}$

(b) $\sum 5n^3$

(c) $\sum \frac{5}{n^4}$

(d) none; the limit comparison test cannot be used

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5. The series $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\sqrt{n}}{n^2-4\sqrt{n}-1}$

(a) converges absolutely (AC)

(b) converges conditionally (CC)

(c) diverges

6. The series $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\sqrt{n}}{n-4\sqrt{n}-1}$

(a) converges absolutely (AC)

(b) converges conditionally (CC)

(c) diverges

7. The series $\displaystyle\sum_{n=0}^{\infty}(-1)^{n}\frac{10^n}{7n!}$

(a) converges absolutely (AC)

(b) converges conditionally (CC)

(c) diverges

8. The series $\displaystyle\sum_{n=2}^{\infty}\left(\frac{2n^2+1}{n^2+5n-6}\right)^n$

(a) converges absolutely (AC)

(b) converges conditionally (CC)

(c) diverges

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9. The interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}(x-1)^n$ is

(a) $[0,1]$

(b) $(0,1)$

(c) $(0,2]$

(d) $[0,2)$

10. A power series representation for the function $f(x)=\dfrac{3}{4-x}$ is

(a) $\sum_{n=0}^{\infty} \frac{3}{4} x^n$

(b) $\sum_{n=0}^{\infty} \left(\frac{3}{4}\right)^{n+1} x^n$

(c) $\sum_{n=0}^{\infty} \frac{3}{4^{n+1}} x^n$

(d) $\sum_{n=0}^{\infty} 3(4-x)^n$

11. The Maclaurin series for the function $f(x) = x^3 \cos(4x^2)$ is

(a) $\sum_{n=0}^{\infty} \frac{(-16)^n}{(2n)!} x^{4n+3}$

(b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n+3}$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} (4x^2)^n$

(d) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{4n^2+3}$

12. The equation of the line tangent to the curve $\begin{array}{c} x = e^{\sqrt{t}} \\ y = t - \ln t^2 \end{array}$ at the point corresponding to $t = 4$ is

(a) $y = \frac{2}{e^2} x + 4 - \ln 16$

(b) $y = \frac{1}{2} x + 4 - \ln 16 - \frac{1}{2} e^2$

(c) $y = \frac{e^2}{4} x + 4 - \ln 16 - \frac{1}{4} e^4$

(d) $y = \frac{2}{e^2} x + 2 - \ln 16$

13. The length of the curve $\begin{array}{c} x = \cos^2 t \\ y = \cos t \end{array}$ is

(a) $\int_0^{2\pi} \sqrt{\sin^2 2t + \sin^2 t}\, dt$

(b) $\int_0^{\pi} \sqrt{\sin^2 2t + \sin^2 t}\, dt$

(c) $\int_0^{2\pi} \sqrt{1 + 14 \sec^2 t}\, dt$

(d) $\int_0^{\pi} \sqrt{1 + 14 \sec^2 t}\, dt$

**Fill-In.**

1. $\sum_{n=1}^{\infty} \frac{5^n}{n^2+1} (x+3)^{n-1}$ is a power series centered at ______ .

2. The radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} x^n$ is ______ .

3. *Circle the best answer. On the line, indicate one valid test that can be applied to get your answer. You may choose from the following list:*

- divergence test
- $p$-series test
- geometric series test
- integral test
- direct comparison test
- limit comparison test
- alternating series test
- ratio test
- root test

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{4}{n^3}$ ( **converges** | **diverges** ) Test: ______________

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^3 - 1}{3n^2}$ ( **converges** | **diverges** ) Test: ______________

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{3\sqrt{n}}{n^2 - 3n + 1}$ ( **converges** | **diverges** ) Test: ______________

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\tan^{-1}(n)}$ ( **converges** | **diverges** ) Test: ______________

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{10^n}{(5n)!}$ ( **converges** | **diverges** ) Test: ______________

**Graph.** *More accuracy = more points!*

Let $C$ be the curve $\begin{array}{c} x = \cos t \\ y = \sin t \cos t \end{array}$.

(a) Eliminate the parameter to find a Cartesian equation of $C$.

(b) Find the point(s) on the curve where the tangent is vertical.

(c) Find the point(s) on the curve where the tangent is horizontal.

(d) Find equation(s) of the tangent(s) to $C$ at the point $(0, 0)$.

(e) Sketch a graph of $C$, labeling the features found in parts (b)-(d).

**Work and Answer.** *You must show all relevant work to receive full credit.*

1. Find the sum of the series $\displaystyle\sum_{n=-1}^{\infty} \frac{2 \cdot 3^n}{4^{n-1}}$.

2. Find the sum of the series $\displaystyle\sum_{n=2}^{\infty} \frac{n+1}{n^3 - n}$.

3. Determine whether the series $\sum_{n=1}^{\infty} \frac{3^n \sin n}{n!}$ is absolutely convergent (AC), conditionally convergent (CC), or divergent.

4. (a) Find a power series representation for the function $f(x) = \ln(2 + 3x)$.
   (b) Find the interval of convergence.

5. (a) Write the Taylor series for the function $f(x) = \sqrt{x}$ centered at 1.
   (b) Find the radius of convergence.
   (c) Estimate $\sqrt{1.4}$ using the first three terms of the Taylor series.

6. Estimate $\int_0^1 e^{x^2}\,dx$ using the first two terms of the Maclaurin series expansion.

7. Estimate $\int_0^1 \sin x^2\,dx$ using the first two terms of the Maclaurin series expansion.

8. Find the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n\,\pi^{2n+1}}{(2n+1)!}$.

9. Find the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n\,\pi^{2n}}{4^n\,(2n)!}$.

10. Find the sum of the series $\sum_{n=2}^{\infty} \frac{5^n}{n!}$.

11. Find the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{(\sqrt{3})^{2n+1}\,(2n+1)}$.

12. Set up, **but do not evaluate,** an integral for the length of the curve $\begin{array}{l} x = \frac{1}{3}t^3 \\ y = \cos t \end{array}$ with $0 \le t \le \frac{\pi}{2}$.

13. For the curve $\begin{array}{c} x = 1 + \tan t \\ y = \cos 2t \end{array}$, find $\frac{dy}{dx}$.

Some kind of **BONUS.**