Burger

M76

DAY 3

Computing Definite Integral using Riemann sums.

ex1.

Evaluate $\displaystyle\int_{-2}^{1} 2x\,dx$.

For this we divide $[-2,1]$ into 'n' subintervals:

$$\Delta x = \frac{1-(-2)}{n} = \boxed{\frac{3}{n} = \Delta x}$$

It is most convenient to use right endpoints:

$$x_k = a + k\Delta x$$
$$= \boxed{-2 + \frac{3k}{n}}$$

So by definition:

$$\int_{-2}^{1} 2x\,dx = \lim_{\substack{n\to\infty \\ \text{or} \\ \Delta x \to 0}} \sum_{k=1}^{n} f\left(-2+\frac{3k}{n}\right)\cdot\frac{3}{n}$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} 2\left(-2+\frac{3k}{n}\right)\bullet\frac{3}{n} = \lim_{n\to\infty} \frac{6}{n}\sum_{k=1}^{n} -2+\frac{3k}{n}$$

$$= \lim_{n\to\infty} \frac{6}{n}\left[\sum_{k=1}^{n} -2 + \frac{3}{n}\sum_{k=1}^{n} k\right]$$

$$= \lim_{n\to\infty} \frac{6}{n}\left[-2n + \frac{3}{n}\left(\frac{n(n+1)}{2}\right)\right]$$

$$\uparrow$$

$$\sum_{i=1}^{n} k = \frac{n(n+1)}{2}$$

$$= \lim_{n\to\infty} -12 + \frac{6}{n}\cdot\frac{3}{\cancel{n}}\cdot\frac{\cancel{n}(n+1)}{2}$$

$$= \lim_{n\to\infty} -12 + \lim_{n\to\infty}\frac{9(n+1)}{n} = \lim_{n\to\infty} -12 + \lim_{n\to\infty} 9 + \lim_{n\to\infty}\frac{9}{n}$$

$$\frac{n+1}{n} \to 1 \qquad \frac{9}{n} \to 0$$

$$= -12 + 9 = -3$$

Ck: $\int_{-2}^{1} 2x\,dx = \frac{2x^2}{2}\Big|_{-2}^{1} = x^2\Big|_{-2}^{1}$

Fund. Theor. Calc.

$$= (1)^2 - (-2)^2 = 1 - 4 = -3 \checkmark$$

ck with F.C.C.

ex. 2

$$\int_{-1}^{1} x^2\,dx = \frac{x^3}{3}\Big|_{-1}^{1} = \frac{1}{3} - \frac{(-1)^3}{3}$$

$$= \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

let $\Delta x = \frac{b-a}{n} = \frac{1-(-1)}{n} = \frac{2}{n}$

And $x_k = -1 + \frac{2k}{n}$

$$\int_{-1}^{1} x^2\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f\left(-1+\frac{2k}{n}\right)\cdot\frac{2}{n}$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \left(-1+\frac{2k}{n}\right)^2\cdot\frac{2}{n}$$

$$= \lim_{n\to\infty} \frac{2}{n}\cdot\sum_{k=1}^{n} 1 - \frac{4k}{n} + \frac{4k^2}{n^2}$$

$$= \lim_{n\to\infty} \frac{2}{n}\left(\sum_{k=1}^{n} 1 - \frac{4}{n}\sum_{k=1}^{n} k + \frac{4}{n^2}\sum_{k=1}^{n} k^2\right)$$

$$= \lim_{n\to\infty} \frac{2}{n}\left(n - \frac{4\cdot n(n+1)}{n\cdot 2} + \frac{4}{n^2}\cdot\frac{n(n+1)(2n+1)}{6}\right)$$

$$= \lim_{n\to\infty} \frac{2}{n}\cdot n - \frac{4(n+1)}{n} + \frac{2}{n}\cdot\frac{2}{3}\cdot\frac{(n+1)(2n+1)}{n}$$

$$= \lim_{n\to\infty} 2 - 4 - \frac{4}{n} + \frac{4}{3}\left(\frac{2n^2+3n+1}{n^2}\right) = -2 + \frac{8}{3}$$

$$= -\frac{6}{3} + \frac{8}{3} = \frac{2}{3}$$

you try:-

① $\int_{4}^{10} 6\,dx$

② $\int_{-2}^{3} x\,dx$

③ $\int_{-1}^{1} x^3\,dx$

④ $\int_{1}^{2} x^2+1\,dx$