Student: _____ **Instructor:** Lance Burger **Assignment:** 2.5
Date: _____ **Course:** Spring 2017 Math 75 - Burger

1. Evaluate the following limit.

$$\lim_{x\to\infty}\left(3+\frac{3}{x^2}\right)$$

Select the correct answer below and, if necessary, fill in the answer box within your choice.

- **A.** $\lim_{x\to\infty}\left(3+\frac{3}{x^2}\right)=$ _____ (Type an integer or a simplified fraction.)
- **B.** The limit does not exist.

2. Find the limit.

$$\lim_{x\to\infty}\frac{\sin 11x}{14x}$$

Select the correct choice below and fill in any answer boxes in your choice.

- **A.** $\lim_{x\to\infty}\frac{\sin 11x}{14x}=$ _____ (Simplify your answer.)
- **B.** The limit does not exist.

3. Evaluate the following limit.

$$\lim_{x\to\infty}\frac{8+3x+3x^2}{x^2}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** $\lim_{x\to\infty}\frac{8+3x+3x^2}{x^2}=$ _____
- **B.** The limit does not exist.

4. Evaluate the following limit.

$$\lim_{x\to\infty}\left(5+\frac{174}{x}+\frac{\sin^4 x^3}{x^2}\right)$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** $\lim_{x\to\infty}\left(5+\frac{174}{x}+\frac{\sin^4 x^3}{x^2}\right)=$ _____
- **B.** The limit does not exist.

5. Find the limit of the polynomial p(x) as x approaches $-\infty$.

$p(x) = 3x^3 - 8x^2 + 1$

The limit of p(x) as x approaches $-\infty$ is [ ].

---

6. Evaluate $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ for the following rational function. Then give the horizontal asymptote of f, if any.

$$f(x) = \frac{5x^4 - 3}{x^5 + 7x^3}$$

Evaluate $\lim_{x\to\infty} f(x)$. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** $\lim_{x\to\infty} \frac{5x^4 - 3}{x^5 + 7x^3} =$ _____ (Simplify your answer.)
- **B.** The limit does not exist and is neither ∞ nor $-\infty$.

Evaluate $\lim_{x\to-\infty} f(x)$. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** $\lim_{x\to-\infty} \frac{5x^4 - 3}{x^5 + 7x^3} =$ _____ (Simplify your answer.)
- **B.** The limit does not exist and is neither ∞ nor $-\infty$.

Give the horizontal asymptote of f, if any. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The horizontal asymptote is _____.
  (Type an equation. Simplify your answer.)
- **B.** There is no horizontal asymptote.

---

7. Suppose p/q is a rational function where the degree of p is 1 greater than the degree of q. Using polynomial long division, p/q can be written as $\frac{p(x)}{q(x)} = mx + b + \frac{r(x)}{s(x)}$ where r/s is a rational function with the property $\frac{r(x)}{s(x)} \to 0$ as $x \to \pm\infty$. This fact implies that $\frac{p(x)}{q(x)} \approx mx + b$ when x is large. The line $y = mx + b$ is an oblique (or slant) asymptote of p/q. Complete parts **(a)** through **(c)** for the function $f(x) = \frac{x^2 - 1}{x + 7}$.

(a) Use polynomial long division to find the oblique asymptote of f.

Choose the correct answer below.

- **A.** $y = x - 1$
- **B.** $y = x - 7$
- **C.** $y = x + 1$
- **D.** $y = x + 7$

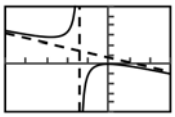
(b) Find the vertical asymptote of f. Select the correct choice below, and, if necessary, fill in the answer box to complete your choice.

- **A.** The vertical asymptote of f is $x =$ _____. (Type an integer or a fraction.)
- **B.** There are no vertical asymptotes.

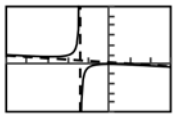
(c) Graph f and all of its asymptotes with a graphing utility.

Choose the correct graph below.

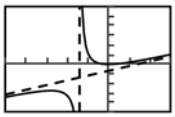
- **A.**



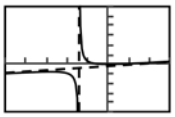
- **B.**



- **C.**



- **D.**



The window setting for all graphs is [-25,15] by [-50,60].

8. Complete the following steps for the given function.

a. Use polynomial long division to find the oblique asymptote of f.
b. Find the vertical asymptotes of f.
c. Graph f and all of its asymptotes with a graphing utility.

$$f(x) = \frac{6x^3 + 5x^2 + 4x + 6}{x^2 + 3}$$

---

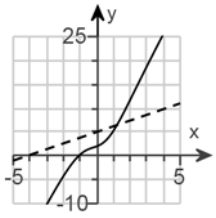
a. The oblique asymptote is $y =$ [ ].

b. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

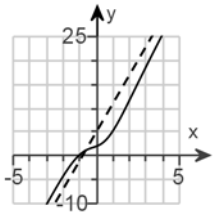
- **A.** The vertical asymptotes are $x =$ _____.
  (Type an integer or a decimal. Use a comma to separate answers as needed.)
- **B.** There are no vertical asymptotes.

c. Graph the function f and all of its asymptotes using a graphing utility. Choose the correct graph below.

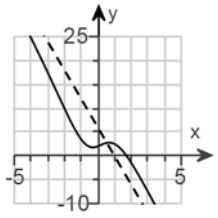
A.



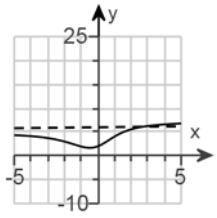
B.



C.



D.



---

9. Consider the function $f(x) = \dfrac{2x^3 + 7x^2 + 6x}{x^3 + 2x^2}$.

(a) Evaluate $\lim\limits_{x \to \infty} f(x)$ and $\lim\limits_{x \to -\infty} f(x)$, then identify the horizontal asymptotes.

(b) Find the vertical asymptote. For the vertical asymptote $x = a$, evaluate $\lim\limits_{x \to a^-} f(x)$ and $\lim\limits_{x \to a^+} f(x)$.

(a)

$\lim\limits_{x \to \infty} f(x) =$ ☐ $\lim\limits_{x \to -\infty} f(x) =$ ☐

Identify the horizontal asymptotes. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ○ **A.** The horizontal asymptote(s) is/are $y =$ _____.
  (Use a comma to separate answers as needed.)
- ○ **B.** There are no horizontal asymptotes.

(b) Find the vertical asymptote. For the vertical asymptote $x = a$, evaluate $\lim\limits_{x \to a^-} f(x)$ and $\lim\limits_{x \to a^+} f(x)$. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ○ **A.** The vertical asymptote is $x =$ _____. The limits at this vertical asymptote are $\lim\limits_{x \to a^-} f(x) =$ _____ and $\lim\limits_{x \to a^+} f(x) =$ _____.
- ○ **B.** There is no vertical asymptote.

---

10. Consider the function $f(x) = \dfrac{2x^4 + 2x^3 - 40x^2}{x^4 - 41x^2 + 400}$.

a. Evaluate $\lim\limits_{x\to\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$, and then identify the horizontal asymptotes.

b. Find the vertical asymptotes. For each asymptote $x = a$, evaluate $\lim\limits_{x\to a^-} f(x)$ and $\lim\limits_{x\to a^+} f(x)$.

---

a. $\lim\limits_{x\to\infty} f(x) =$ [ ] (Simplify your answer.)

$\lim\limits_{x\to-\infty} f(x) =$ [ ] (Simplify your answer.)

Identify all the horizontal asymptotes. Select the correct choice below and fill in any answer boxes within your choice.

- **A.** $y =$ _____
  (Use a comma to separate answers as needed.)
- **B.** There are no horizontal asymptotes.

b. Identify all the vertical asymptotes. Select the correct choice below and fill in any answer boxes within your choice.

- **A.** There are two vertical asymptotes.
  The leftmost one is at $x =$ _____. The limits are $\lim\limits_{x\to a^-} f(x) =$ _____ and $\lim\limits_{x\to a^+} f(x) =$ _____.
  The rightmost one is at $x =$ _____. The limits are $\lim\limits_{x\to a^-} f(x) =$ _____ and $\lim\limits_{x\to a^+} f(x) =$ _____.
- **B.** There is one vertical asymptote.
  The asymptote is at $x =$ _____. The limits are $\lim\limits_{x\to a^-} f(x) =$ _____ and $\lim\limits_{x\to a^+} f(x) =$ _____.
- **C.** There are no vertical asymptotes.

---

11. Consider the function $f(x) = \dfrac{x^2 - 9}{x(x-3)}$. Complete parts a and b.

a. Evaluate $\lim\limits_{x\to\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$, and then identify the horizontal asymptotes.

$\lim\limits_{x\to\infty} \dfrac{x^2-9}{x(x-3)} =$ ☐ $\lim\limits_{x\to-\infty} \dfrac{x^2-9}{x(x-3)} =$ ☐

Identify the horizontal asymptotes. Select the correct choice below and, if necessary, fill in all the answer boxes to complete your choice.

- **A.** The function has a horizontal asymptote at $y =$ _____.
- **B.** The function has horizontal asymptotes at $y =$ _____ and $y =$ _____. (Use ascending order.)
- **C.** The function has no horizontal asymptote.

b. Find the vertical asymptotes. For each vertical asymptote $x = a$, evaluate $\lim\limits_{x\to a^-} f(x)$ and $\lim\limits_{x\to a^+} f(x)$.

Select the correct choice below, and, if necessary, fill in all the answer boxes to complete your choice.

- **A.** The function has a vertical asymptote at $x =$ _____. The limits at this vertical asymptote are $\lim\limits_{x\to a^-} f(x) =$ _____ and $\lim\limits_{x\to a^+} f(x) =$ _____.
- **B.** The vertical asymptote at $x =$ _____ has the limits $\lim\limits_{x\to a^-} f(x) =$ _____ and $\lim\limits_{x\to a^+} f(x) =$ _____. The vertical asymptote at $x =$ _____ has the limits $\lim\limits_{x\to a^-} f(x) =$ _____ and $\lim\limits_{x\to a^+} f(x) =$ _____. (Use ascending order.)
- **C.** The function has no vertical asymptote.

12. For the function $f(x) = \dfrac{\sqrt{4x^2 + 3x + 2} - 3}{x - 1}$, find the following.

(a) Evaluate $\lim\limits_{x \to \infty} f(x)$ and $\lim\limits_{x \to -\infty} f(x)$, and then identify the horizontal asymptotes.

(b) Find the vertical asymptote. For the vertical asymptote $x = a$, evaluate $\lim\limits_{x \to a^-} f(x)$ and $\lim\limits_{x \to a^+} f(x)$.

(a)

$\lim\limits_{x \to \infty} f(x) =$ ☐ $\lim\limits_{x \to -\infty} f(x) =$ ☐

Identify the horizontal asymptotes. Choose the correct answer below.

- ○ **A.** The horizontal asymptotes is/ are $y =$ _____.
  (Use a comma to separate answers as needed.)
- ○ **B.** There are no horizontal asymptotes.

(b) Find the vertical asymptote. For the vertical asymptote $x = a$, evaluate $\lim\limits_{x \to a^-} f(x)$ and $\lim\limits_{x \to a^+} f(x)$. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ○ **A.** The vertical asymptote is $x =$ _____. The limits at this vertical asymptote are $\lim\limits_{x \to a^-} f(x) =$ _____ and $\lim\limits_{x \to a^+} f(x) =$ _____.
- ○ **B.** There is no vertical asymptote.

1. A. $\lim_{x\to\infty}\left(3+\frac{3}{x^2}\right)=$ **3** (Type an integer or a simplified fraction.)

---

2. A. $\lim_{x\to\infty}\frac{\sin 11x}{14x}=$ **0** (Simplify your answer.)

---

3. A. $\lim_{x\to\infty}\frac{8+3x+3x^2}{x^2}=$ **3**

---

4. A. $\lim_{x\to\infty}\left(5+\frac{174}{x}+\frac{\sin^4 x^3}{x^2}\right)=$ **5**

---

5. $-\infty$

---

6. A. $\lim_{x\to\infty}\frac{5x^4-3}{x^5+7x^3}=$ **0** (Simplify your answer.)

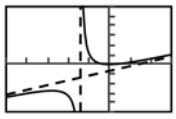
A. $\lim_{x\to-\infty}\frac{5x^4-3}{x^5+7x^3}=$ **0** (Simplify your answer.)

A. The horizontal asymptote is **y = 0**. (Type an equation. Simplify your answer.)

---

7. B. $y=x-7$

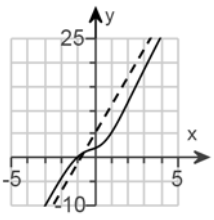
A. The vertical asymptote of f is $x=$ **−7**. (Type an integer or a fraction.)

C. 

---

8. $6x+5$

B. There are no vertical asymptotes.

B. 

---

9. 2

2

A. The horizontal asymptote(s) is/are y = **2** .(Use a comma to separate answers as needed.)

A.

The vertical asymptote is x = **0** . The limits at this vertical asymptote are $\lim_{x \to a^-} f(x) =$ **$-\infty$** and $\lim_{x \to a^+} f(x) =$ **∞** .

---

10. 2

2

A. y = **2** (Use a comma to separate answers as needed.)

A. There are two vertical asymptotes.

The leftmost one is at x = **-4** . The limits are $\lim_{x \to a^-} f(x) =$ **∞** and $\lim_{x \to a^+} f(x) =$ **$-\infty$** .

The rightmost one is at x = **5** . The limits are $\lim_{x \to a^-} f(x) =$ **$-\infty$** and $\lim_{x \to a^+} f(x) =$ **∞** .

---

11. 1

1

A. The function has a horizontal asymptote at y = **1** .

A.

The function has a vertical asymptote at x = **0** . The limits at this vertical asymptote are $\lim_{x \to a^-} f(x) =$ **$-\infty$** and $\lim_{x \to a^+} f(x) =$ **∞** .

---

12. 2

-2

A. The horizontal asymptotes is/ are y = **2, -2** . (Use a comma to separate answers as needed.)

B. There is no vertical asymptote.
