Math 250/M2_Study Guide/Fall ' 13 'POTENTIAL' TYPES OF TEST QUESTIONS: Burger

1. CONTINUED FRACTIONS: Can you fill in a rectangle diagram? Can you simplify an infinite continued fraction as a square root of a positive integer?

Chapter 3

2. Name a field having exactly 243 elements.

3. -Can you find the multiplicative inverse of $[x^2 + 2x]$ in $\mathbb{Z}_3[x]/(x^3 + 1)$.

   -Can you find inverses for all non-zero elements in F? If not, find a zero divisor pair.

4. Let $f(x) = x^3 + x + 1 \in \mathbb{Z}_2[x]$ and :
   a. Show $f(x)$ is irreducible.
   b. Denote α as a root of $f(x)$ in an extension field $E \supset \mathbb{Z}_2$.
   c. Find a field $F \supset E$ in which $f(x)$ factors into linear terms. List linear factors of f, denoting other needed root as β.
   d. How many elements in F?

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Chapter 5

5. There will be a proof involving the following concepts: rings; ring isomorphism; integral domain, so know those definitions.

6. Show that:
   a. (2, x) in $Z[x]$ is not a principal ideal. See example 11, p. 297.
   b. (3) in $Z[x]$ is not a maximal ideal.

7. #5 in 5.6/questions ... show $2+3\sqrt{-5}$ is irreducible in $Z[\sqrt{-5}]$ but not prime. See answer to #5 on page 417. Note that in a PID, irreducible implies prime and Z[i] is a PID, while $Z[\sqrt{-5}]$ is not.

8. Is $8-i$ irreducible in Z[i]? Is it prime? Is ⟨8-i⟩ a maximal ideal in Z[i]?

9. -Carry out the division algorithm in Z[i] for 4+5i and 6i-2. Be able to solve a Diophantine equation in Z[i] as I did in class recently.
   -or I could instead ask you to show an ideal of Z[i] is principal, such as I=$\{a(4+5\mathrm{i})+\mathrm{b}(6\mathrm{i}-2)\colon \mathrm{a,b} \in \mathrm{Z[i]}\}$, in which case the gcd would be the generator.

10. By Theorem 7, Z[i] a PID implies all non-zero, non-unit elements can factor into prime, irreducible elements, up to associates. Completely factor an element of Z[i], such as $-5+5i$