Practice Final Exam (a)

Math 142

PART I

1. (28 points total) Evaluate the following integrals.

(a)
$$\int x^2\, e^{-x^3}\, dx$$

(b)
$$\int_1^e \frac{\sqrt{\ln x}}{x}\, dx$$

(c)
$$\int_1^3 x\,(x^2+1)^{3/2}\, dx$$

(d)
$$\int \frac{\sin t}{(\cos t + 1)^2}\, dt$$

2. (8 points) Find the average value of $f(x) = x^3 + x$ on the interval $[1, 3]$.

3. (26 points) Let R be the region in the first quadrant bounded by the curves $y = x^2$ and $y = 4x - x^2$. Calculate:

(a) The area of R.

(b) The volume obtained by rotating R about the x-axis (slice method is recommended).

(c) The volume obtained by rotating R about the y-axis (shell method is recommended).

4. (12 points) A rectangular swimming pool is 50 m long, 15 m wide, and 3 m deep. The depth of the water is 2 m. How much work is required to pump all of the water out over the side ? (Use $\rho = 1000$ kg/m^3 for the density of the water and $g = 10$ m/s^2 for the acceleration of gravity.)

5. (10 points) A particle moves along a line with velocity function $v(t) = t^2 - 2t - 8$. Find:

(a) The displacement during the time interval $[1, 6]$;

(b) The distance traveled during the time interval $[1, 6]$.

6. (16 points) If $f(x) = x^3 - 3x^2 + 5$,

(a) Find the intervals on which $f(x)$ is increasing.

(b) Find the intervals on which $f(x)$ is decreasing.

(c) Find the local maxima of $f(x)$.

(d) Find the local minima of $f(x)$.

(e) Find the intervals on which $f(x)$ is concave up.

(f) Find the intervals on which $f(x)$ is concave down .

(g) Find the inflection points of $f(x)$.

(h) Sketch the graph of $f(x)$.

PART II

7. (32 points) Evaluate the following integrals:

(a)

$$\int x^2 \sin 4x \, dx$$

(b)

$$\int_1^e x^5 \ln x \, dx$$

(c)

$$\int_3^4 \frac{1}{x^2 - 3x + 2} \, dx$$

(d)

$$\int \frac{1}{(x-1)(x^2+1)} \, dx$$

8. (16 points) Evaluate the following integrals:

(b) (use a trigonometric substitution)

$$\int \sqrt{9 + x^2} \, dx$$

9. (16 points) Evaluate the following integrals or show that they are divergent:

(a)

$$\int_2^{10} \frac{1}{\sqrt[3]{x-2}} \, dx$$

(b)

$$\int_0^{\infty} x \, e^{-2x} \, dx$$

10. (8 points) Set up Simpson's Rule sum with $n = 4$ subdivisions for the integral:

$$\int_0^4 \sqrt{1+x^3}\, dx.$$

11. (12 points) Find the length of the curve $y = \frac{4}{3}x^{3/2}$, $0 \leq x \leq 2$.

12. (16 points) Find the area of the surface obtained by rotating the curve $y = x^3$, $0 \leq x \leq 1$ about the x-axis.